12-1 Inverse Variation

Check Skills You'll Need

remember $y = kx$ or $k = \frac{y}{x}$

Suppose y varies directly with x. Find each constant of variation. (k)

1. $y = 5x$

$y = kx$

$k = 5$

2. $y = -7x$

$k = {}^{-}7$

3. $3y = x$

$y = \frac{1}{3}x$

$k = \frac{1}{3}$

4. $0.25y = x$

$y = 4x$

$k = 4$

Write an equation of the direct variation that includes the given point.

5. (2, 4)

$k = \frac{y}{x}$

$k = \frac{4}{2}$

$y = 2x$

6. (3, 1.5)

$k = \frac{1.5}{3} = \frac{1}{2}$

$y = \frac{1}{2}x$

7. (–4, 1)

$k = \frac{-1}{4}$

$y = \frac{-1}{4}x$

8. (–5, –2)

$k = \frac{2}{5}$

$y = \frac{2}{5}x$

Investigation: Inverse Variation

Suppose you are part of a volunteer crew constructing affordable housing. Building a house requires a total of 160 workdays. For example, a crew of 20 people can complete a house in 8 days.

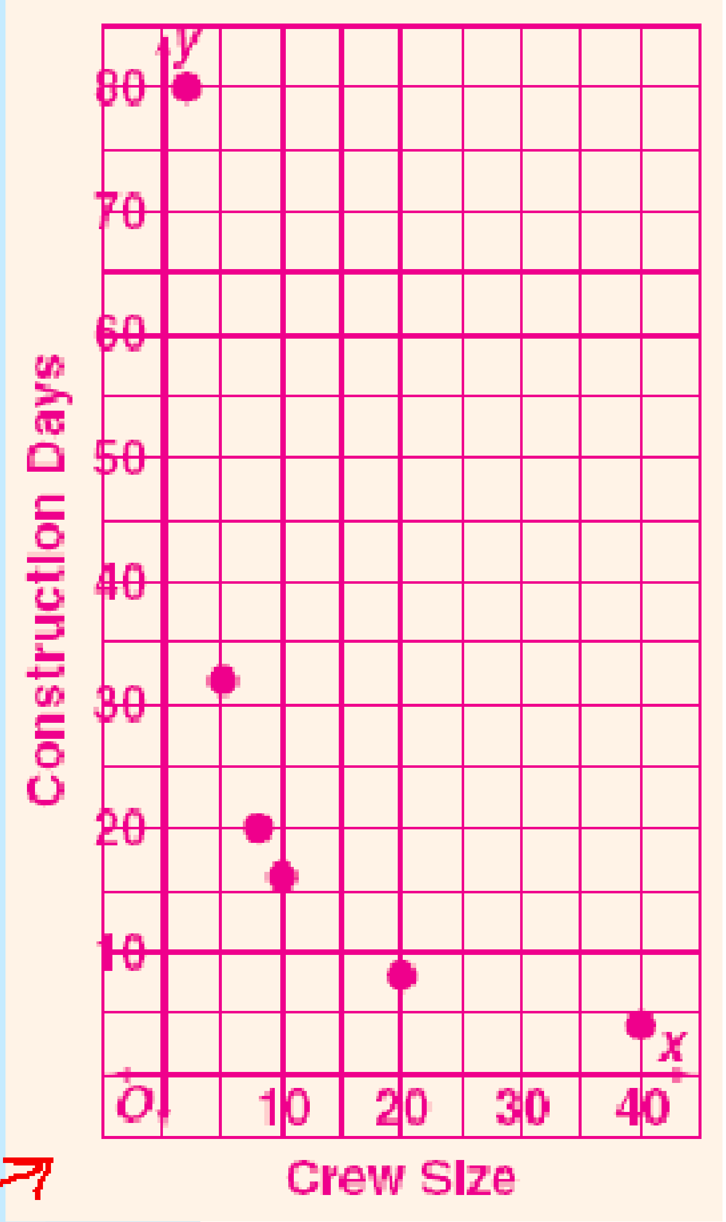
1. How long should it take a crew of 40 people?

2. Copy and complete the table.

As x increases, y decreases.

Crew size (x)	Construction Days (y)	Total Workdays
2	80	160
5	32	160
8	20	160
10	16	160
20	8	160
40	4	160

3. Graph the (x, y) data in the table above.



4. Describe what happens to construction time as the crew size increases.

In the table, the total number of workdays remains the same. The number of construction days decreases as the number of people on the crew increases. The relationship of construction days and crew size is an inverse variation.

Definition **Inverse Variation**

An equation in the form $xy = k$ or $y = \frac{k}{x}$, where $k \neq 0$, is an **inverse variation.**

The **constant of variation** is k.

Inverse variations have graphs with the same general shape. You can see from the graph at the right how the constant of variation k affects the graph of $xy = k$.

If you know the values of x and y for one point on the graph of an inverse variation, you can use the point to find the constant of variation k and the equation of the inverse variation.

1 EXAMPLE Writing an Equation Given a Point

Suppose y varies inversely with x and $y = 7$ when $x = 5$. Write an equation for the inverse variation.

$xy = k$	**Use the general form of an inverse variation.**
$5(7) = k$	**Substitute 5 for *x* and 7 for *y*.**
$35 = k$	**Multiply to solve for *k*.**
$xy = 35$	**Write an equation. Substitute 35 for *k* in *xy* = *k*.**

The equation of the inverse variation is $xy = 35$, or $y = \frac{35}{x}$.

Check Understanding

1 Suppose y varies inversely with x and $y = 9$ when $x = 2$. Write an equation for the inverse variation.

$k = 9 \cdot 2 = 18$ ... so $xy = 18$ or $y = \frac{18}{x}$

Suppose (x_1, y_1) and (x_2, y_2) are two ordered pairs of an inverse variation. Each ordered pair of an inverse variation has the same product k, that is $x_1 \cdot y_1 = k$ and $x_2 \cdot y_2 = k$. So $x_1 \cdot y_1 = x_2 \cdot y_2$.

2 EXAMPLE Finding the Missing Coordinate

The points (3, 8) and (2, y) are two points on the graph of an inverse variation. Find the missing value.

$x_1 \cdot y_1 = x_2 \cdot y_2$	**Use the equation $x_1 \cdot y_1 = x_2 \cdot y_2$ since you know coordinates but not the constant of variation.**
$3(8) = 2(y_2)$	**Substitute 3 for x_1, 8 for y_1, and 2 for x_2.**
$24 = 2(y_2)$	**Simplify.**
$12 = y_2$	**Solve for y_2.**

The missing value is 12. The point (2, 12) is on the graph of the inverse variation that includes the point (3, 8).

Check Understanding

2 Each pair of points is on the graph of an inverse variation. Find the missing value.

a. $(3, y)$ and $(5, 9)$ **15**

$3y = 5 \cdot 9$

$3y = 45$

b. $(75, 0.2)$ and $(x, 3)$ **5**

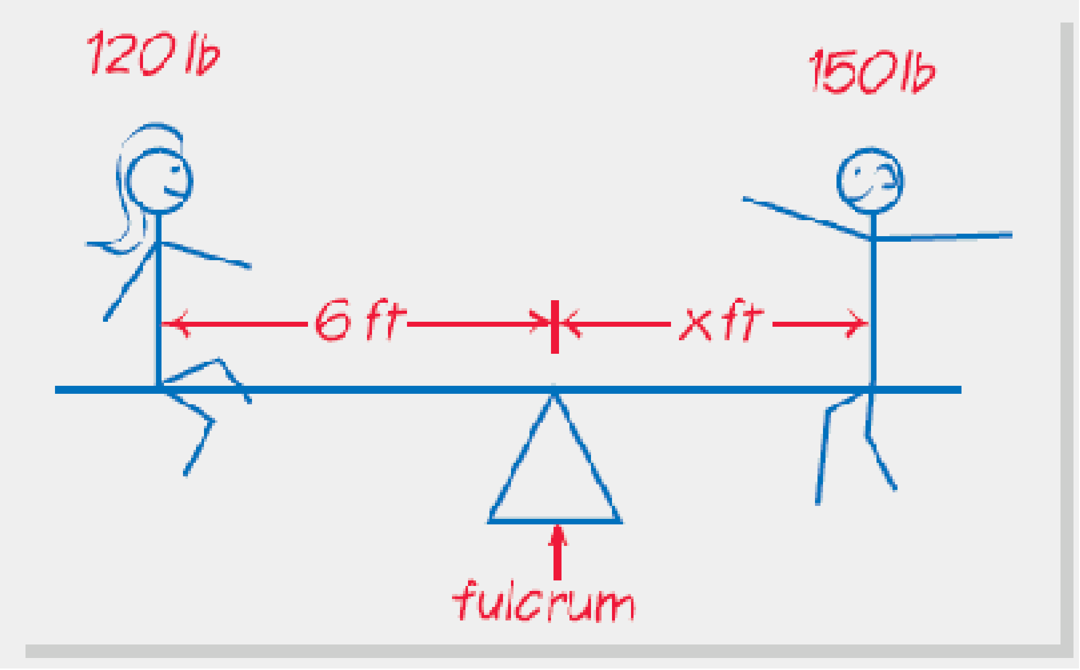
$75(0.2) = 3x$

$15 = 3x$

3 EXAMPLE Real-World Problem Solving

Physics The weight needed to balance a lever varies inversely with the distance from the fulcrum to the weight. Where should Julio, who weighs 150 lb, sit to balance the lever?

$$120 \cdot 6 = 150x$$
$$720 = 150x$$
$$x = 4.8'$$



Julio should sit 4.8 feet from the fulcrum to balance the lever.

Check Understanding

3 a. **Physics** A 100-lb weight is placed 4 ft from a fulcrum. How far from the fulcrum should a 75-lb weight be placed to balance the lever? $5.\overline{3}$ ft

b. An 80-lb weight is placed 9 ft from a fulcrum. What weight should you put 6 ft from the fulcrum to balance the lever? 120 lb

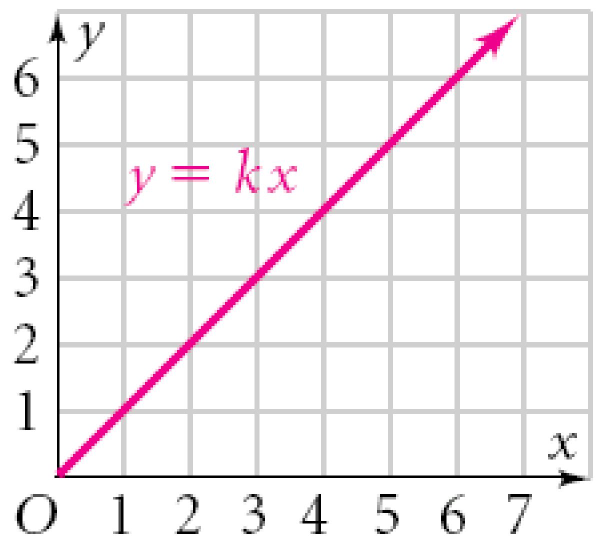
a) $100 \cdot 4 = 75x$
$400 = 75x$

b) $80 \cdot 9 = 6x$
$720 = 6x$

Recall that a direct variation is an equation in the form $y = kx$. This summary will help you recognize and use direct and inverse variations.

Summary **Direct and Inverse Variation**

Direct Variation

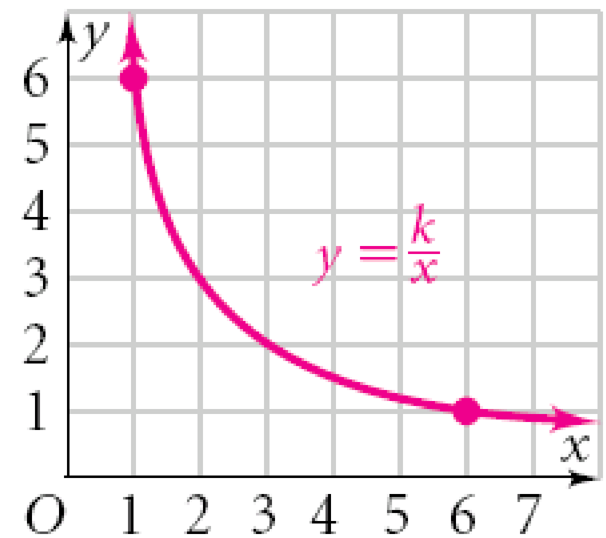


y varies directly with x.

y is directly proportional to x.

The ratio $\frac{y}{x}$ is constant.

Inverse Variation



y varies inversely with x.

y is inversely proportional to x.

The product xy is constant.

4 EXAMPLE Determining Direct or Inverse Variation

Do the data in each table represent a *direct variation* or an *inverse variation*? For each table, write an equation to model the data.

a.

x	y
2	5
4	10
10	25

Look for a constant of variation: Does $\mathbf{k} = {}^{\mathbf{y}}/_{\mathbf{x}}$ or $\mathbf{xy}$?

$2 \cdot 5 \neq 4 \cdot 10 \neq 10 \cdot 25$ $\quad k \neq xy$

$\frac{5}{2} = \frac{10}{4} = \frac{25}{10}$ $\quad k = \frac{y}{x} = 2.5$

direct variation

b.

x	y
5	20
10	10
25	4

$5 \cdot 20 = 10 \cdot 10 = 25 \cdot 4$ $\quad k = xy = 100$

inverse variation

Check Understanding

4 Determine whether the data in each table represent a direct variation or an inverse variation. Write an equation to model the data in each table.

a.

x	y
3	12
6	6
9	4

$k = xy = 36$

inverse

$xy = 36$

b.

x	y
3	12
5	20
8	32

$k = \frac{y}{x} = 4$

direct

$y = 4x$

5 EXAMPLE Real-World Problem Solving

Explain whether each situation represents a direct variation or an inverse variation.

a. **Carpooling** The cost of \$20 worth of gasoline is split among several people.

The cost per person times the number of people equals the total cost of the gasoline. Since the total cost is a constant product of \$20, this is an inverse variation.

b. **School Supplies** You buy several markers for 70¢ each.

The cost per marker times the number of markers equals the total cost of the markers. Since the ratio $\frac{\text{cost}}{\text{marker}}$ is constant at 70¢ each, this is a direct variation.

Check Understanding

5 Explain whether each situation represents a direct variation or an inverse variation.

a. You are in a discount store. All sweaters are on sale for \$15 each.

b. You walk 5 miles each day. Your speed and time vary from day to day.

a.) Buy more sweaters, pay more money. DIRECT

b.) Increasing speed will decrease your time. INVERSE

Assignment:

p.640, # 1-29

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Student Edition Answers

pages 640–642 Exercises

1. $xy = 18$

2. $xy = 2$

3. $xy = 56$

4. $xy = 1.5$

5. $xy = 24$

6. $xy = 7.7$

7. $xy = 2$

8. $xy = 0.5$

9. $xy = 0.06$

10. 8

11. 15

12. 6

13. 7

14. 3

15. 130

16. 12

17. 96

18. 3125

19. 2

20. $\frac{1}{6}$

21. 20

22. 3 h

23. $13.\overline{3}$ mi/h

24. direct variation; $y = 0.5x$

25. inverse variation; $xy = 60$

26. inverse variation; $xy = 72$

27. Direct variation; the ratio $\frac{\text{cost}}{\text{pound}}$ is constant at \$1.79.

28. Inverse variation; the total number of slices is constant at 8.

29. Inverse variation; the product of the length and width remains constant with an area of 24 square units.

30. 32; $xy = 32$

31. 1.1; $rt = 1.1$

32. 2.5; $xy = 2.5$