

12-3

Simplifying Rational Expressions

 Check Skills You'll Need

Write each fraction in simplest form.

1. $\frac{8}{2} = 4$

2. $-\frac{15}{24} = -\frac{5}{8}$

3. $\frac{25}{35} = \frac{5}{7}$

Factor each quadratic expression.

4. $x^2 + x - 12 = (x+4)(x-3)$

5. $x^2 + 6x + 8 = (x+2)(x+4)$

6. $x^2 - 2x - 15 = (x-5)(x+3)$

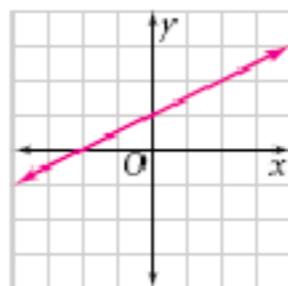
7. $x^2 + 8x + 16 = (x+4)^2$

8. $x^2 - x - 12 = (x-4)(x+3)$

9. $x^2 - 7x + 12 = (x-4)(x-3)$

**Linear function**

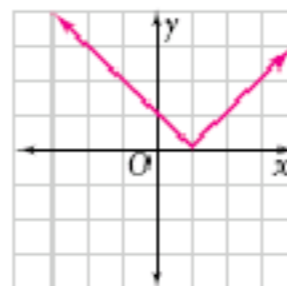
$$y = mx + b$$

slope = m y-intercept = b

The greatest exponent is 1.

Absolute value function

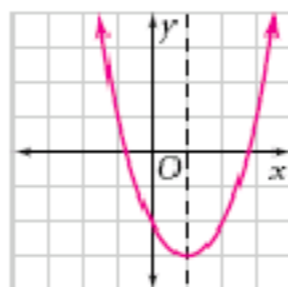
$$y = |x - a| + b$$

shift $y = |x|$ horizontally a unitsshift $y = |x|$ vertically b unitsvertex at (a, b)

The greatest exponent is 1.

Quadratic function

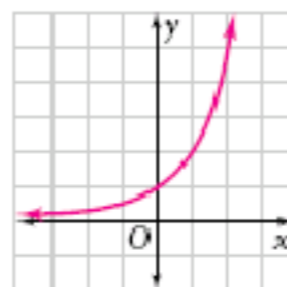
$$y = ax^2 + bx + c$$

parabola with axis of symmetry at $x = -\frac{b}{2a}$

The greatest exponent is 2.

Exponential function

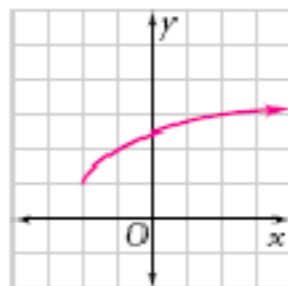
$$y = ab^x$$

growth for $b > 1$ decay for $0 < b < 1$

The variable is the exponent.

Radical function

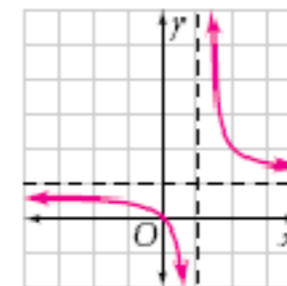
$$y = \sqrt{x - b} + c$$

shift $y = \sqrt{x}$ horizontally b unitsshift $y = \sqrt{x}$ vertically c units

The variable is under the radical.

Rational function

$$y = \frac{a}{x - b} + c$$

vertical asymptote at $x = b$ horizontal asymptote at $y = c$

The variable is in the denominator.

Fractions like $\frac{5}{9}$, $\frac{7}{12}$, and $\frac{1}{2}$ are rational numbers. An expression with a variable in the denominator is a **rational expression**. Here are some examples of rational expressions.

$$\frac{1}{x} \qquad \frac{x+2}{x-3} \qquad \frac{x^2-5}{x^2-10x+25}$$

$x \neq 0$ $x \neq 3$ $(x-5)(x-5)$
 $x \neq 5$

Of course, the value of the expression in the denominator cannot be zero, since division by zero is undefined. For the rest of this chapter, assume that the values of the variables that make the denominator zero are excluded from the domain.

Like rational numbers, a rational expression is in simplest form if the numerator and denominator have no common factors except 1. For example, $\frac{z+5}{10z}$ is in simplest form since neither 10 nor z is a factor of $z+5$.

1**EXAMPLE****Simplifying a Rational Expression**Simplify $\frac{6x + 12}{x + 2}$.

$$\frac{\cancel{6(x+2)}}{\cancel{(x+2)}} = 6$$

 **Check Understanding**
1 Simplify each expression.

a. $\frac{15b}{25b^2}$

$$\frac{3}{5b}$$

b. $\frac{12c^2}{3c + 6}$

$$\frac{12c^2}{3(c+2)}$$

$$\frac{4c^2}{c+2}$$

c. $\frac{4m - 2}{2m - 1}$

$$\frac{\cancel{2(2m-1)}}{\cancel{(2m-1)}}$$

$$2$$

d. $\frac{20 + 4t}{t + 5}$

$$\frac{4(\cancel{5+t})}{\cancel{(t+5)}}$$

$$4$$

Recall that you learned to factor quadratic expressions in Lessons 9-5 and 9-6. You may need to factor a quadratic expression to simplify a rational expression.

2 EXAMPLE Simplifying a Rational Expression

Simplify $\frac{2x - 12}{x^2 - 7x + 6} = \frac{2\cancel{(x-6)}}{(x-1)\cancel{(x-6)}} = \frac{2}{x-1}$

4

Do you remember this? $\rightarrow (2a^2 + 4a) + (a + 2)$

✓ Check Understanding $2a(a+2) + 1(a+2)$

2 Simplify each expression.

a. $\frac{3x + 12}{x^2 - x - 20}$

$$\frac{3\cancel{(x+4)}}{(x-5)\cancel{(x+4)}}$$

$$\frac{3}{x-5}$$

b. $\frac{2z - 2}{z^2 - 4z + 3}$

$$\frac{2\cancel{(z-1)}}{(z-3)\cancel{(z-1)}}$$

$$\frac{2}{z-3}$$

c. $\frac{8a + 16}{2a^2 + 5a + 2}$

$$\frac{8\cancel{(a+2)}}{(2a+1)\cancel{(a+2)}}$$

$$\frac{8}{2a+1}$$

d. $\frac{c^2 - c - 6}{c^2 + 5c + 6}$

$$\frac{\cancel{(c-3)}\cancel{(c+2)}}{(c+3)\cancel{(c+2)}}$$

$$\frac{c-3}{c+3}$$

3**EXAMPLE****Recognizing Opposite Factors**Simplify $\frac{5x - 15}{9 - x^2}$.

$$\frac{5(x-3)}{(3+x)(3-x)} = \frac{5\cancel{(x-3)}}{-1(3+x)\cancel{(x-3)}}$$

$$= \frac{-5}{x+3}$$

✓ Check Understanding**3** Simplify each expression.

a. $\frac{x-4}{4-x}$

$$\frac{\cancel{(x-4)}}{-1\cancel{(x-4)}}$$

$$-1$$

b. $\frac{8-m}{m^2-64}$

$$\frac{-1\cancel{(8-m)}}{(m+8)\cancel{(m-8)}}$$

$$\frac{-1}{m+8}$$

c. $\frac{8-4r}{r^2+2r-8}$

$$\frac{-1 \cdot 4\cancel{(2-r)}}{(r+4)\cancel{(r-2)}}$$

$$\frac{-4}{r+4}$$

d. $\frac{2c^2-2}{3-3c^2}$

$$\frac{-1 \cdot 2\cancel{(c^2-1)}}{3\cancel{(1-c^2)}}$$

$$\frac{-2}{3}$$



Assignment: p. 654,
1-33 (odd), 37

I ♥ U
IRONMAN!

pages 654–656 Exercises

37. -3 is not in the domain of $\frac{x^2 - 9}{x + 3}$

1. $\frac{2a + 3}{4}$

2. $\frac{1}{7x}$

3. $\frac{1}{3}$

4. $\frac{1}{2}$

5. $3x$

6. $\frac{x + 2}{x^2}$

7. $\frac{2}{3}$

8. $\frac{2}{b + 4}$

9. $\frac{1}{m - 7}$

10. $\frac{w}{w - 7}$

11. $\frac{a + 1}{5}$

12. $\frac{m + 3}{m + 2}$

13. $\frac{c - 4}{c + 3}$

14. $b + 3$

15. $\frac{1}{m - 2}$

16. -1

17. $\frac{-4}{t + 1}$

18. -2

19. $-\frac{1}{2}$

20. $-\frac{1}{v + 5}$

21. $-\frac{1}{w - 4}$

22. 36 min

23. 13 min

24. 13 min

25. $\frac{2r - 1}{r + 5}$

26. $\frac{7z + 2}{z - 1}$

27. $\frac{5t - 4}{3t - 1}$

28. $\frac{4a^2}{2a - 1}$

29. $\frac{3(z + 4)}{z^3}$

30. $\frac{2s + 1}{s^2}$

31. $-\frac{2a + 1}{a + 3}$

32. $\frac{4 + 3m}{m - 7}$

33. $\frac{-c(3c + 5)}{5c + 4}$