Dividing Polynomials

Check Skills You'll Need

Write each polynomial in standard form. (Decending order of x)

1. $9a - 4a^2 + 1$ $-4a^2 + 9a + 1$

2. $3x^2 - 6 + 5x - x^3$ $-x^3 + 3x^2 + 5x - 6$

3. $-2 + 8t$ $8t - 2$

Find each product.

4. $(2x + 4)(x + 3)$ $2x^2 + 10x + 12$

5. $(-3n - 4)(n - 5)$ $-3n^2 + 11n + 20$

6. $(3a^2 + 1)(2a - 7)$ $6a^3 - 21a^2 + 2a - 7$

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial divisor.

1 EXAMPLE Dividing a Polynomial by a Monomial

Divide $8x^3 + 4x^2 - 12x$ by $2x^2$.

$$4 \quad \frac{8x^3}{2x^2} + \frac{24x^2}{2x^2} - \frac{12x}{2x^2} \qquad 6$$

$$4x + 2 - \frac{6}{x}$$

Check Understanding

1 Divide.

a. $(3m^3 - 6m^2 + m) \div 3m^2$

$$\frac{3m^3}{3m^2} - \frac{6m^2}{3m^2} + \frac{m}{3m^2}$$

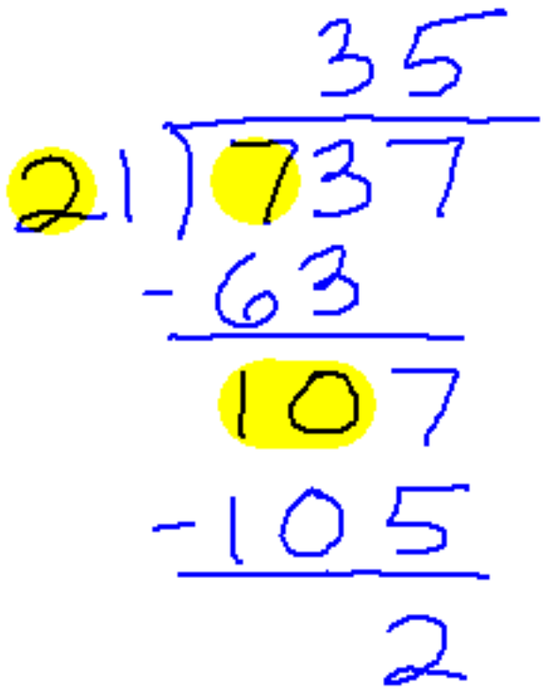
$$m - 2 + \frac{1}{3m}$$

b. $(8t^5 + 16t^3 - 4t^2 + 2t) \div 4t^2$

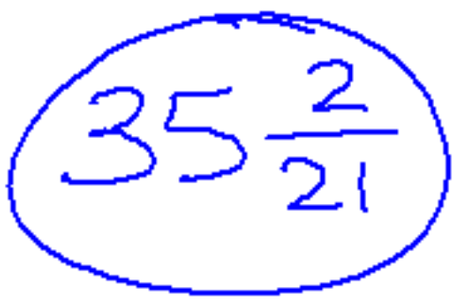
$$\frac{8t^5}{4t^2} + \frac{16t^3}{4t^2} - \frac{4t^2}{4t^2} + \frac{2t}{4t^2}$$

$$2t^3 + 4t - 1 + \frac{1}{2t}$$

The process of dividing a polynomial by a binomial is similar to long division. For example, consider dividing 737 by 21.



1. **Divide:** 21 can go into 73 about 3 times.
2. **Multiply** 3 × 21, then **subtract** from 73.
3. **Bring down** the 7. Divide: 107 ÷ 21 ≈ 5.
4. Multiply 5 × 21, and then subtract from 107.
5. The remainder is 2.



You can summarize the process for long division as

"Divide, multiply, subtract, bring down, and repeat as necessary."

In the division above, the answer is written as a mixed number: $35\frac{2}{21}$ means $35 + \frac{2}{21}$.
In dividing polynomials, write the answer as quotient + $\frac{\text{remainder}}{\text{divisor}}$.

When the divisor and dividend are in standard form, divide the first term of the dividend by the first term of the quotient.

2 EXAMPLE Dividing a Polynomial by a Binomial

Divide $2y^2 + 3y - 40$ by $y + 5$.

$$y+5 \overline{)\,2y^2 + 3y - 40}$$

$$2y - 7 - \frac{5}{y+5}$$

$$-2y^2 + -10y$$

$$-7y - 40$$

$$+7y + 35$$

$$-5$$

Check Understanding 2 Divide.

a. $(2b^2 - b - 3) \div (b + 1)$

$$b+1 \overline{)\,2b^2 - b - 3}$$

$2b - 3$

$-(2b^2 + 2b)$

$-3b - 3$

$+(3b + 3)$

0

no remainder!

b. $(6m^2 - 5m - 7) \div (2m + 1)$

$$2m+1 \overline{)\,6m^2 - 5m - 7}$$

$3m - 4 - \frac{3}{2m+1}$

$-(6m^2 + 3m)$

$-8m - 7$

$+(8m + 4)$

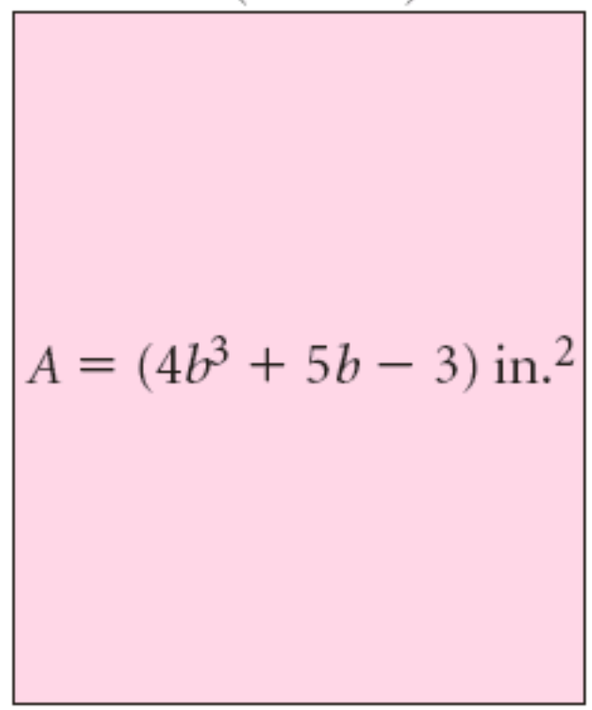
-3

When the dividend is in standard form and a power is missing, add a term of that power with 0 as its coefficient. For example, rewrite $4b^3 + 5b - 3$,

as $4b^3 + 0b^2 + 5b - 3$.

3 EXAMPLE Dividing Polynomials With a Zero Coefficient

Geometry The width and area of a rectangle are shown in the figure. What is the length?



Length

$$2b-1\overline{)\,4b^3+0b^2+5b-3}$$

$2b^2 + b + 3$

$-4b^3 + 2b^2$

$2b^2 + 5b$

$-2b^2 + b$

$6b - 3$

$-6b + 3$

$L = 2b^2 + b + 3$

Check Understanding 3 Divide.

a. $(t^4 + t^2 + t - 3) \div (t - 1)$

$$
\begin{array}{r}
t^3 + t^2 \quad +1 \\
t-1\,\overline{)\,t^4 + 0t^3 + t^2 + t - 3} \\
\underline{-t^4 + t^3} \\
t^3 + t^2 \\
\underline{-t^3 + t^2} \\
t - 3 \\
\underline{-t + 1} \\
-2
\end{array}
$$

$$t^3 + t^2 + 1 - \frac{2}{t-1}$$

b. $(c^3 - 4c + 12) \div (c + 3)$

$$
\begin{array}{r}
c^2 - 3c + 5 \\
c+3\,\overline{)\,c^3 + 0c^2 - 4c + 12} \\
\underline{-c^3 + 3c^2} \\
-3c^2 - 4c \\
\underline{+3c^2 + 9c} \\
5c + 12 \\
\underline{-5c + 15} \\
-3
\end{array}
$$

$$c^2 - 3c + 5 - \frac{3}{c+3}$$

To use the process for long division, write any divisor or dividend in standard form before you begin to divide.

4 EXAMPLE Reordering Terms and Dividing Polynomials

Divide $-3x + 4 + 9x^2$ by $1 + 3x$.

$3x+1\overline{)\,9x^2-3x+4}$

Check Understanding

4 Divide.

a. $(10x - 1 + 8x^2) \div (1 + 2x)$

$2x+1\overline{)\,8x^2+10x-1}$

b. $(9 - 6a^2 - 11a) \div (3a - 2)$

$3a-2\overline{)\,-6a^2-11a+9}$

$4x + 3 - \frac{4}{2x+1}$

$-2a - 5 - \frac{1}{3a-2}$

Assignment: **p.656, CPQ 1-10,**
p.661, # 47-73 (odd),
p.664, # 1-17 (odd).

Summary **Dividing a Polynomial by a Polynomial**

Step 1 Arrange the terms of the dividend and divisor in standard form.

Step 2 Divide the first term of the dividend by the first term of the divisor. This is the first term of the quotient.

Step 3 Multiply the first term of the quotient by the divisor and place the product under the dividend.

Step 4 Subtract this product from the dividend.

Step 5 Bring down the next term.

Repeat Steps 2–5 as necessary until the degree of the remainder is less than the degree of the divisor.

p.656 CPQ 1-10, p.661 47-73(odd), p.664 1-17(odd)

47. $\frac{9m^2(m+1)}{2}$

48. 1

49. $\frac{x}{y+5}$

50. $\frac{-(2a+3b)(a+2b)}{(5a+b)(2a-3b)}$

51. $\frac{m-2}{2m(m-1)}$

52. $\frac{x(x-2)}{2(x-1)}$

53. $\frac{1}{(w+2)(w+3)}$

54. B

55. G

56. D

57. G

59. $\frac{b-5}{2}$

60. $\frac{3}{4k}$

61. $\frac{7}{3}$

62. $\frac{q^4}{4}$

63. $\frac{5t^2-9}{8}$

64. $\frac{m^2}{2-3m}$

65. $\frac{1}{2a^2-3}$

66. $\frac{2z+3}{z+1}$

67. $\frac{2c-9}{2c+8}$

68. 8.2

69. 5.3

70. 5

71. 11

72. 0.2

73. 7.1

p.656 CPQ 1-10, p.661 47-73(odd), p.664 1-17(odd)

Student Edition Answers

pages 664–666 Exercises

1. $x^4 - x^3 + x^2$

2. $3x^4 - \frac{2}{x}$

3. $3c^2 + 2c - \frac{1}{3}$

4. $n^2 - 18n + 3$

5. $4 - \frac{16}{q}$

6. $-t^3 + 2t^2 - 4t + 5$

7. $x - 3$

8. $2t + 9 + \frac{16}{t - 3}$

9. $n - 1$

10. $y - 3 + \frac{8}{y + 2}$

11. $3x - 1$

12. $-2q - 10 + \frac{22}{2q + 1}$

13. $5t - 50$

14. $2w^2 + 2w + 5 - \frac{10}{w - 1}$

15. $b^2 - 3b - 1 + \frac{3}{3b - 1}$

16. $c^2 - \frac{1}{c - 1}$

17. $t^2 - 2t - 2$

18. $n^2 - 2n - 21 - \frac{8}{n + 2}$

19. $(r^2 + 5r + 1)$ cm

20. $(4c^2 - 8c + 16)$ ft

21. $b + 12 + \frac{1}{b + 4}$