2.11 Factor $y = ax^2 + bx + c$

Math I

Goal • Graph general quadratic functions.

VOCABULARY

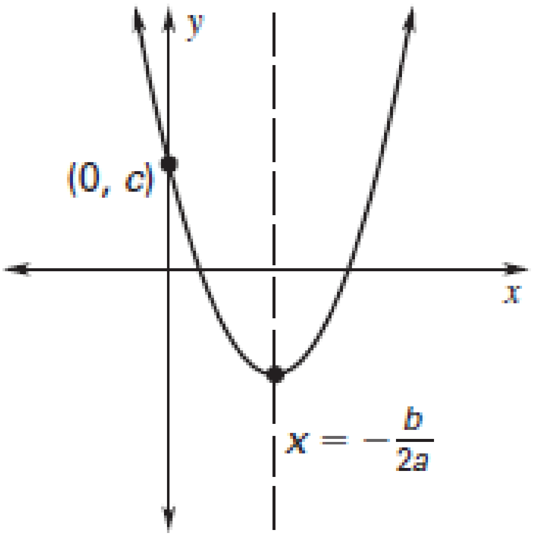
Minimum value When $a > 0$, the minimum value of the function $y = ax^2 + bx + c$ is the y-coordinate of the vertex.

Maximum value When $a < 0$, the maximum value of the function $y = ax^2 + bx + c$ is the y-coordinate of the vertex.

PROPERTIES OF THE GRAPH OF A QUADRATIC FUNCTION

The graph of $y = ax^2 + bx + c$ is a parabola that:

- opens **up** if $a > 0$ and opens **down** if $a < 0$.
- is narrower than the graph of $y = x^2$ if $|a|$ **>** 1 and wider if $|a|$ **<** 1.
- has an axis of symmetry of $x = -\frac{b}{2a}$.
- has a vertex with an x-coordinate of $-\frac{b}{2a}$.
- has a y-intercept of **c**. So, the point (**0** , **c**) is on the parabola.



MINIMUM AND MAXIMUM VALUES

For $y = ax^2 + bx + c$, the y-coordinate of the vertex is the **minimum** value of the function if a **$>$** 0 and the **maximum** value of the function if a **$<$** 0.

Consider the function $y = -2x^2 + 16x - 15$.

a. Find the axis of symmetry of the graph of the function.

b. Find the vertex of the graph of the function.

Solution

a. For the function $y = -2x^2 + 16x - 15$, $a =$ __-2__ and $b =$ __16__.

$$x = -\frac{b}{2a} = -\frac{16}{2(-2)} = 4$$

The axis of symmetry is $x =$ __4__.

b. The x-coordinate of the vertex is $-\frac{b}{2a}$, or __4__. To find the y-coordinate, substitute __4__ for x in the function and find y.

$$y = -2(4)^2 + 16(4) - 15 = 17$$

The vertex is (__4__, __17__).

Tell whether the function $f(x) = 5x^2 - 20x + 17$ has a *minimum value* or a *maximum value*. Then find the minimum or maximum value.

Solution

Because $a =$ 5 and 5 > 0, the parabola opens up and the function has a minimum value. To find the maximum value, find the vertex.

$x = -\frac{b}{2a} = -\frac{-20}{2(5)} = 2$ — **The x-coordinate is $-\frac{b}{2a}$.**

$f(2) = 5(2)^2 - 20(2) + 17$ — **Substitute 2 for x.**

$= -3$ — **Simplify.**

The minimum value of the function is f(2) = -3.

Checkpoint Find the axis of symmetry and the vertex of the graph of the function.

1. $y = 3x^2 + 18x + 5$

a = 3 and b = 18

$$x = -\frac{18}{2(3)} = -\frac{18}{6} = -3$$

Axis of symmetry is x = -3

Vertex: (– 3, – 22)

$y = 3(-3)^2 + 18(-3) + 5$

$y = 3(9) - 54 + 5$

$y = 27 - 54 + 5$

$y = -22$

Checkpoint Find the axis of symmetry and the vertex of the graph of the function.

2. $y = \frac{1}{4}x^2 - 4x + 7$

a = ¼ and b = – 4

$$x = -\frac{-4}{2(1/4)} = -\frac{-4}{2/4} = \quad \frac{4}{1} \bullet \frac{4}{2} = \quad \frac{16}{2} = 8$$

Axis of symmetry is x = 8

$y = ¼ (8)^2 – 4(8) + 7$

y = ¼(64) – 32 + 7

y = 16 – 32 + 7

y = - 9

Vertex: (8, –9)

3. Tell whether the function $f(x) = -\frac{1}{2}x^2 + 6x + 8$ has a *minimum value* or a *maximum value*. Then find the minimum or maximum value.

Since a = – ½ and – ½ < 0, the parabola opens down.

$|a| = |-½| = ½ < 1$, the parabola is wider than $y = x^2$.

$$x = -\frac{6}{2(-\frac{1}{2})} = -\frac{6}{-1} = 6$$

$f(x) = -½(6)^2 + 6(6) + 8$

$f(x) = -½(36) + 36 + 8$

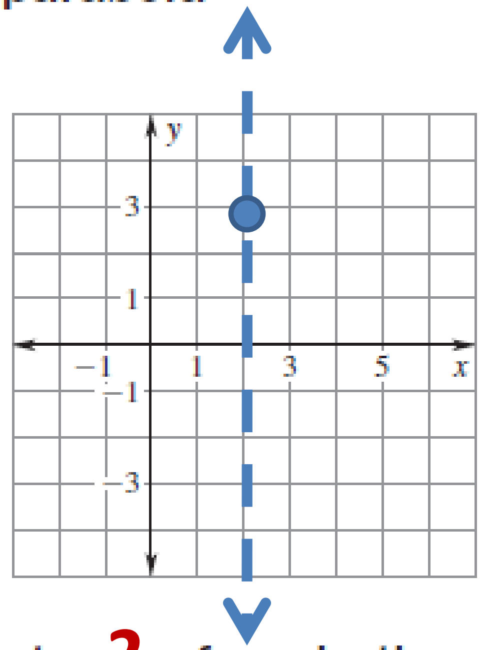
$f(x) = -18 + 36 + 8$

$f(x) = 26$

Since – ½ < 0, the function has a maximum.

Graph $y = -x^2 + 4x - 1$.

Step 1 **Determine** whether the parabola opens up or down. Because a __<__ 0, the parabola opens __down__.



Step 2 **Find** and draw the axis of symmetry:

$x = -\frac{b}{2a} = \underline{-\frac{4}{2(-1)}} = \underline{2}$.

Step 3 **Find** and plot the vertex.

The x-coordinate of the vertex is $\underline{-\frac{b}{2a}}$, or __2__.

To find the y-coordinate, substitute __2__ for x in the function and simplify.

$y = -(\underline{2})^2 + 4(\underline{2}) - 1 = 3$

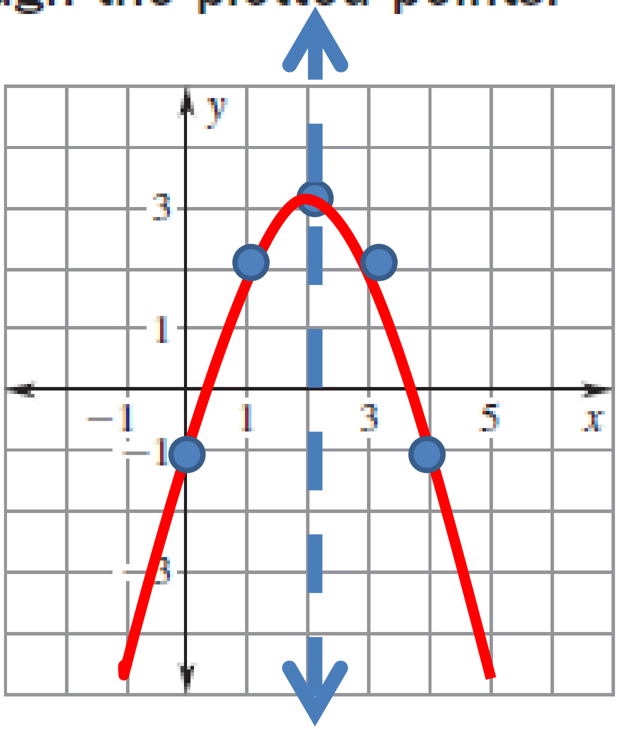
So, the vertex is (__2__, __3__).

Step 4 **Plot** two points. Choose two *x*-values less than the *x*-coordinate of the vertex. Then find the corresponding *y*-values.

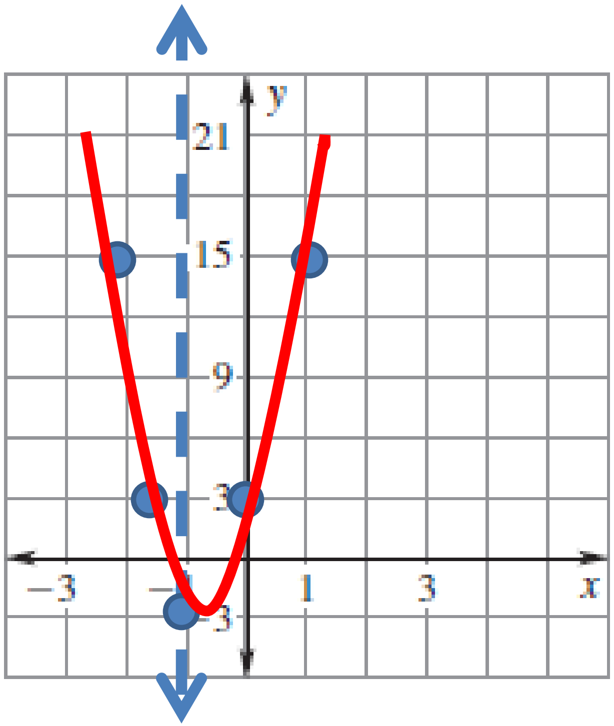
x	1	0
y	2	-1

Step 5 Reflect the points plotted in Step 4 in the axis of symmetry.

Step 6 **Draw** a parabola through the plotted points.



4. Graph the function $y = 4x^2 + 8x + 3$. Label the vertex and axis of symmetry.



$y = 4x^2 + 8x + 3$

$$x = -\frac{8}{2(4)} = -\frac{8}{8} = -1$$

$y = 4(-1)^2 + 8(-1) + 3$

$y = 4(1) + -8 + 3$

$y = -1$

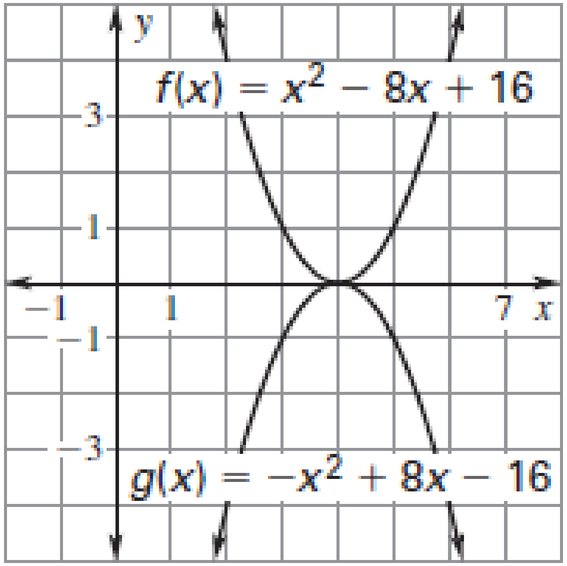
Vertex: (–1, – 1)

Since a = 4 and 4 > 0, the parabola opens up.

x	0	1
y	3	15

Compare the graph of $f(x) = x^2 - 8x + 16$ and $g(x) = -x^2 + 8x - 16$.

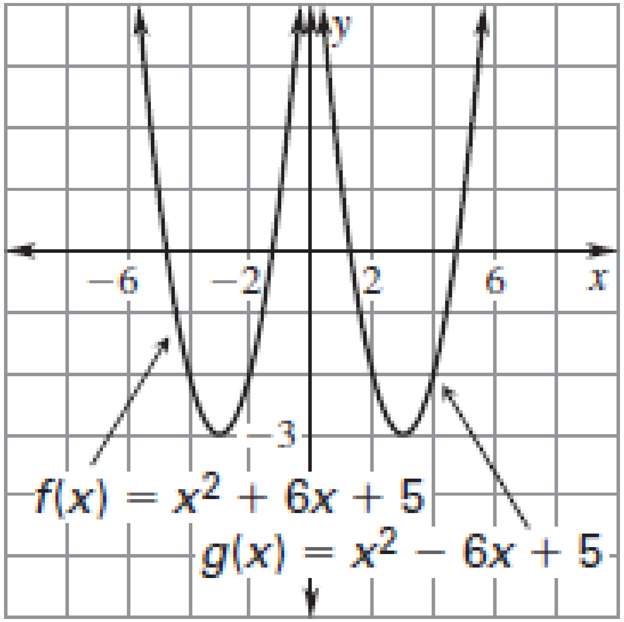
Solution



Consider the **x-axis** as a mirror. The graph of $g(x) = -x^2 + 8x - 16$ is the mirror image of the graph of $f(x) = x^2 - 8x + 16$. So, the graph of $g(x)$ is a **reflection in the x-axis** of the graph of $f(x)$.

5. $f(x) = x^2 + 6x + 5$

$g(x) = x^2 - 6x + 5$



The graph of g(x) is a reflection in the y-axis of the graph of f(x).

6. $f(x) = x^2 + 3x + 4$

$g(x) = -x^2 - 3x - 4$

The graph of g(x) is a reflection in the x-axis of the graph of f(x).