2.9 Factor Polynomials Completely

Math I

2.9 Factor Polynomials Completely

Goal • Factor polynomials completely.

VOCABULARY

Factor by grouping **Factoring a polynomial with four terms by factoring a common monomial from a pair of terms, then looking for a common binomial factor.**

Factor completely **Factoring a polynomial until it is written as a product of unfactorable polynomials with integer coefficients**

Example 1 *Factor out common binomial*

Factor the expression.

a. $3x(x + 2) - 2(x + 2)$

b. $y^2(y - 4) + 3(4 - y)$

Solution

a. $3x(x + 2) - 2(x + 2) = (x + 2)($ **3x – 2** $)$

b. The binomials $y - 4$ and $4 - y$ are **opposites**.
Factor **- 1** from $4 - y$ to obtain a common binomial factor.

$y^2(y - 4) + 3(4 - y) = y^2(y - 4)$ **– 3(y – 4)**

$= (y - 4)$**$(y^2 - 3)$**

Example 2 ***Factor by grouping***

Factor the polynomial.

a. $y^3 + 7y^2 + 2y + 14$ **b.** $y^2 + 2y + yx + 2x$

Solution

a. $y^3 + 7y^2 + 2y + 14 = (\underline{y^3 + 7y^2}) + (\underline{(2y + 14)})$

$= \underline{y^2}(\underline{y + 7}) + \underline{2}(\underline{y + 7})$

$= (\underline{y + 7})(\underline{y^2 + 2})$

b. $y^2 + 2y + yx + 2x = (\underline{y^2 + 2y}) + (\underline{yx + 2x})$

$= \underline{y}(\underline{y + 2}) + \underline{x}(\underline{y + 2})$

$= (\underline{y + 2})(\underline{y + x})$

Example 3 *Factor by grouping*

Factor $x^3 - 12 + 3x - 4x^2$.

Solution

$$x^3 - 12 + 3x - 4x^2 = x^3 - 4x^2 + 3x - 12$$

$$= (x^3 - 4x^2) + (3x - 12)$$

$$= x^2(x - 4) + 3(x - 4)$$

$$= (x^2 + 3)(x - 4)$$

Checkpoint Factor the expression.

1. $5z(z - 6) + 4(z - 6)$	2. $2y^2(y - 1) + 7(1 - y)$
$(5z + 4)(z - 6)$	$2y^2(y - 1) - 7(y - 1)$

$$(2y^2 - 7)(y - 1)$$

3. $x^3 - 4x^2 + 5x - 20$	4. $n^3 + 48 + 6n + 8n^2$

$(x^3 - 4x^2) + (5x - 20)$

$x^2(x - 4) + 5(x - 4)$

$(x^2 + 5)(x - 4)$

$n^3 + 8n^2 + 6n + 48$

$(n^3 + 8n^2) + (6n + 48)$

$n^2(n + 8) + 6(n + 8)$

$(n^2 + 6)(n + 8)$

Example 4 *Factor completely*

Factor the polynomial completely.

a. $x^2 + 3x - 1$

b. $3r^3 - 21r^2 + 30r$

c. $9d^4 - 4d^2$

Solution

a. This polynomial cannot be factored.

b. $3r^3 - 21r^2 + 30r = 3r(r^2 - 7r + 10)$

$= 3r(r - 2)(r - 5)$

c. $9d^4 - 4d^2 = d^2(9d - 4)$ $\quad d^2[(3d)^2 - (2)^2]$

$= d^2(3d - 2)(3d - 2)$

Checkpoint **Factor the expression.**

5. $-2x^3 + 6x^2 + 108x$ $-2x(x^2 - 3x - 54)$ $-2x(x - 9)(x + 6)$	**6.** $12y^4 - 75y^2$ $3y^2(4y^2 - 25)$ $3y^2[(2y)^2 - (5)^2]$ $3y^2(2y - 5)(2y + 5)$

Example 5 *Solve a polynomial equation*

Solve $5x^3 - 25x^2 = -30x$.

Solution

$5x^3 - 25x^2 = -30x$	Write original equation.
$5x^3 - 25x^2$ **+** $30x = 0$	**Add** 30x to each side.
$5x(x^2 - 5x + 6) = 0$	Factor out **5x**.
$5x(x - 3)(x - 2) = 0$	Factor trinomial.
$5x = 0$ *or* $x - 3 = 0$ *or* $x - 2 = 0$	Zero-product property
$x = 0$ $\quad x = 3$ $\quad x = 2$	Solve for *x*.

Checkpoint **Solve the equation.**

7. $2x^3 + 2x^2 = 40x$	**8.** $-4x^3 + 72x = -12x^2$
$2x^3 + 2x^2 - 40x = 0$	$-4x^3 + 12x^2 + 72x = 0$
$2x(x^2 + x - 20) = 0$	$-4x(x^2 - 3x - 18) = 0$
$2x(x + 5)(x - 4) = 0$	$-4x(x + 3)(x - 6) = 0$
$2x = 0$ $x + 5 = 0$ $x - 4 = 0$	$-4x = 0$ $x + 3 = 0$ $x - 6 = 0$
$x = 0$ $x = -5$ $x = 4$	$x = 0$ $x = -3$ $x = 6$