

3.10 Add and Subtract Rational Expressions



Least common denominator of rational expressions
(LCD)

The least common denominator of two or more rational expressions is the product of the factors of the rational expressions with each common factor used only once.



Example 1**Add and subtract with the same denominator**

Find the sum or difference.

a. $\frac{3}{8x} + \frac{4}{8x}$

b. $\frac{2x + 9}{x + 1} - \frac{7}{x + 1}$

Solution

$$\begin{aligned} \text{a. } \frac{3}{8x} + \frac{4}{8x} &= \frac{\boxed{3 + 4}}{8x} \\ &= \frac{\boxed{7}}{\underline{8x}} \end{aligned}$$

Add numerators.

Simplify.

$$\text{b. } \frac{2x + 9}{x + 1} - \frac{7}{x + 1} = \frac{\boxed{(2x + 9) - 7}}{x + 1}$$

Subtract numerators.

$$= \frac{\boxed{2x + 2}}{x + 1}$$

Simplify.

$$= \frac{\boxed{\cancel{2(x + 1)}}}{\boxed{\cancel{(x + 1)}}}$$

Factor and divide out common factor.

$$= \underline{2}$$

Simplify.



$$\begin{aligned} \mathbf{1.} \quad & \frac{x+8}{4x} + \frac{3}{4x} \\ &= \frac{x+8+3}{4x} \\ &= \frac{x+11}{4x} \end{aligned}$$

$$\begin{aligned} \mathbf{2.} \quad & \frac{6x-5}{x} - \frac{2x-5}{x} \\ &= \frac{(6x-5)-(2x-5)}{x} \\ &= \frac{6x-5-2x+5}{x} \\ &= \frac{\cancel{4x}}{\cancel{x}} \\ &= 4 \end{aligned}$$



Find the LCD of the rational expressions.

a. $\frac{1}{3x^3}, \frac{5}{4x^4}$

Solution

a. Find the **least common multiple** of $3x^3$ and $4x^4$.

$$\begin{array}{r} 3x^3 = \quad \mathbf{3} \quad \mathbf{x} \quad \mathbf{x} \quad \mathbf{x} \\ \hline 4x^4 = \quad \mathbf{4} \quad \mathbf{x} \quad \mathbf{x} \quad \mathbf{x} \quad \mathbf{x} \\ \hline \end{array}$$

$$\text{LCM} = \mathbf{3} \quad \mathbf{4} \quad \mathbf{x} \quad \mathbf{x} \quad \mathbf{x} \quad \mathbf{x} = \mathbf{12x^4}$$

The LCD of $\frac{1}{3x^3}$ and $\frac{5}{4x^4}$ is $12x^4$.



Find the LCD of the rational expressions.

b. $\frac{7}{x^2 - 4}, \frac{x + 3}{x^2 + x - 2}$

b. Find the least common multiple of $x^2 - 4$ and $x^2 + x - 2$.

$$x^2 - 4 = \frac{(x + 2)(x - 2)}{1}$$

$$x^2 + x - 2 = \frac{(x + 2)(x - 1)}{1}$$

$$\text{LCM} = \frac{(x + 2)(x - 2)(x - 1)}{1}$$

The LCD of $\frac{7}{x^2 - 4}$ and $\frac{x + 3}{x^2 + x - 2}$ is

$$\frac{(x + 2)(x - 2)(x - 1)}{1}$$



3. $\frac{5}{36x} \cdot \frac{x+2}{4x^3}$

$36x:$ 2 2 3 3 x
 $4x^3:$ 2 2 x x x

LCM: 2 2 3 3 x x x

LCM: $36x^3$



$$4. \frac{7x}{x-8}, \frac{x-1}{x+3}$$

$$x-8: x-8$$

$$x+3: x+3$$

$$\text{LCM: } (x+3)(x-8)$$



Example 3**Add expressions with different denominators**

Find the sum $\frac{1}{3x^3} + \frac{5}{4x^4}$.

Solution

$$\begin{aligned} & \frac{1}{3x^3} + \frac{5}{4x^4} \\ &= \frac{1 \cdot \boxed{4x}}{3x^3 \cdot \boxed{4x}} + \frac{5 \cdot \boxed{3}}{4x^4 \cdot \boxed{3}} \\ &= \frac{\boxed{4x}}{\boxed{12x^4}} + \frac{\boxed{15}}{\boxed{12x^4}} \\ &= \frac{\boxed{4x + 15}}{\boxed{12x^4}} \end{aligned}$$

Rewrite fractions using LCD,
 $12x^4$.

Simplify numerators and
denominators.

Add fractions.



Example 4**Subtract expressions with different denominators**

Find the difference $\frac{x + 1}{x^2 + 5x + 6} - \frac{x - 4}{x^2 - 9}$.

Solution

$$\frac{x + 1}{x^2 + 5x + 6} - \frac{x - 4}{x^2 - 9}$$

$$= \frac{x + 1}{\boxed{x + 2} \boxed{x + 3}} - \frac{x - 4}{\boxed{x - 3} \boxed{x + 3}}$$

$$= \frac{(x + 1) \boxed{(x - 3)}}{\boxed{(x + 2)} \boxed{(x + 3)} \boxed{(x - 3)}} - \frac{(x - 4) \boxed{(x + 2)}}{\boxed{(x + 3)} \boxed{(x - 3)} \boxed{(x + 2)}}$$

$$\frac{(x + 1)(x - 3) - (x - 4)(x + 2)}{(x + 2)(x + 3)(x - 3)} = \frac{x^2 - 2x - 3 - (x^2 - 2x - 8)}{(x + 2)(x + 3)(x - 3)}$$

$$= \frac{x^2 - 2x - 3 - x^2 + 2x + 8}{(x + 2)(x + 3)(x - 3)} = \frac{5}{(x + 2)(x + 3)(x - 3)}$$



$$5. \frac{9}{x-1} - \frac{15}{3x+1}$$

$$\frac{9(3x+1)}{(x-1)(3x+1)} - \frac{15(x-1)}{(x-1)(3x+1)}$$

$$\frac{27x+9-(15x-15)}{(x-1)(3x+1)}$$

$$\frac{27x+9-15x+15}{(x-1)(3x+1)}$$

$$\frac{12x+24}{(x-1)(3x+1)}$$

$$\frac{12(x+2)}{(x-1)(3x+1)}$$



$$6. \frac{12}{5x} + \frac{3x}{x-4}$$

$$\frac{12(x-4)}{(5x)(x-4)} + \frac{3x(5x)}{(5x)(x-4)}$$

$$\frac{12x - 48 + 15x^2}{(5x)(x-4)}$$

$$\frac{15x^2 + 12x - 48}{(5x)(x-4)}$$

$$\frac{3(5x^2 + 4x - 16)}{(5x)(x-4)}$$



$$7. \frac{x-1}{x^2-2x-24} + \frac{4}{x^2-5x-6}$$

$$\frac{x^2-4x-17}{(x+4)(x-6)(x+1)}$$

$$\frac{x-1}{(x+4)(x-6)} - \frac{4}{(x-6)(x+1)}$$

$$\frac{(x-1)(x+1)}{(x+4)(x-6)(x+1)} - \frac{4(x+4)}{(x+4)(x-6)(x+1)}$$

$$\frac{x^2+1x-1x-1}{(x+4)(x-6)(x+1)} - \frac{(4x+16)}{(x+4)(x-6)(x+1)}$$

$$\frac{x^2-1-4x-16}{(x+4)(x-6)(x+1)}$$



$$8. \frac{x+2}{x^2+2x-15} - \frac{x-6}{x^2+4x-21}$$

$$\frac{2(5x+22)}{(x+5)(x-3)(x+7)}$$

$$\frac{x+2}{(x+5)(x-3)} - \frac{x-6}{(x-3)(x+7)}$$

$$\frac{(x+2)(x+7)}{(x+5)(x-3)(x+7)} - \frac{(x-6)(x+5)}{(x+5)(x-3)(x+7)}$$

$$\frac{(x^2+7x+2x+14) - (x^2+5x-6x-30)}{(x+5)(x-3)(x+7)}$$

$$\frac{x^2+9x+14-x^2-5x+6x+30}{(x+5)(x-3)(x+7)}$$

$$\frac{10x+44}{(x+5)(x-3)(x+7)}$$

