3.13 Use Sequences

VOCABULARY

Sequence

A function whose domain is a set of consecutive whole numbers.

Terms

The values in the range of a sequence.

Example 1 *Write terms of a sequence*

Write the first six terms of the sequence. Identify the domain and range.

a. $a_n = 3n - 1$

Solution

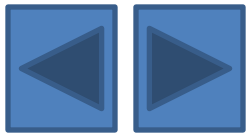
a. $a_1 = 3(\underline{1}) - 1 = \underline{2}$ $\quad a_2 = 3(\underline{2}) - 1 = \underline{5}$

$a_3 = 3(\underline{3}) - 1 = \underline{8}$ $\quad a_4 = 3(\underline{4}) - 1 = \underline{11}$

$a_5 = 3(\underline{5}) - 1 = \underline{14}$ $\quad a_6 = 3(\underline{6}) - 1 = \underline{17}$

Domain: 1, 2, 3, 4, 5, 6

Range: 2, 5, 8, 11, 14, 17



Example 1 *Write terms of a sequence*

Write the first six terms of the sequence. Identify the domain and range.

b. $a_n = 32\left(-\frac{1}{2}\right)^{n-1}$

b. $a_1 = 32\left(-\frac{1}{2}\right)^{1-1} =$ 32

$a_2 = 32\left(-\frac{1}{2}\right)^{2-1} =$ –16

$a_3 = 32\left(-\frac{1}{2}\right)^{3-1} =$ 8

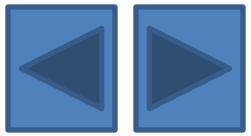
$a_4 = 32\left(-\frac{1}{2}\right)^{4-1} =$ –4

$a_5 = 32\left(-\frac{1}{2}\right)^{5-1} =$ 2

$a_6 = 32\left(-\frac{1}{2}\right)^{6-1} =$ –1

Domain: 1, 2, 3, 4, 5, 6

Range: 32, –16, 8, –4, 2, –1



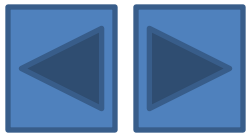
Example 2 *Write rules for sequences*

Describe the pattern, write the next term, and write a rule for the *n*th term of the sequence (a) 1, 4, 9, 16, . . . and (b) −7, −14, −21, −28, ...

Solution

a. You can write the terms as 1^2, 2^2, 3^2, 4^2, ... The next term is $a_5 = 5^2 = 25$. A rule for the *n*th term is $a_n = n^2$.

b. You can write the terms as $-7(1)$, $-7(2)$, $-7(3)$, $-7(4)$, ... The next term is $a_5 = -7(5) = -35$. A rule for the *n*th term is $a_n = -7n$.



Checkpoint **Write the first six terms of the sequence. Identify the domain and range.**

1. $a_n = n + 9$

$a_1 = 1 + 9 = 10$

$a_2 = 2 + 9 = 11$

$a_3 = 3 + 9 = 12$

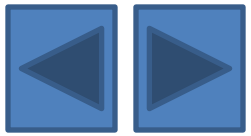
$a_4 = 4 + 9 = 13$

$a_5 = 5 + 9 = 14$

$a_6 = 6 + 9 = 15$

Domain: 1, 2, 3, 4, 5, 6

Range: 10, 11, 12, 13, 14, 15



Checkpoint **Write the first six terms of the sequence. Identify the domain and range.**

2. $a_n = (-3)^n$

Domain: 1, 2, 3, 4, 5, 6

Range: –3, 9, –27, 81, –243, 729

$a_1 = (-3)^1 = -3$

$a_2 = (-3)^2 = (-3)(-3) = 9$

$a_3 = (-3)^3 = (-3)(-3)(-3) = -27$

$a_4 = (-3)^4 = (-3)(-3)\ (-3)(-3) = 81$

$a_5 = (-3)^5 = (-3)(-3)(-3)(-3)(-3) = -243$

$a_6 = (-3)^6 = (-3)(-3)(-3)(-3)(-3)\ (-3) = 729$

3. For the sequence 0, −3, −8, −15, . . ., describe the pattern, write the next term, and write a rule for the *n*th term.

You can write the terms as:

$-1^2 + 1, -2^2 + 1, -3^2 + 1, -4^2 + 1, \ldots;$

Notice that -1^2 is "the opposite of 1 squared", NOT "negative 1 squared".

$$a_5 = -5^2 + 1 = -25 + 1 = -24$$

$$a_n = -n^2 + 1$$

Band A band is arranged in 5 rows. The first 3 rows are shown at the right. Write a rule for the number of musicians in each row. Then graph the sequence.

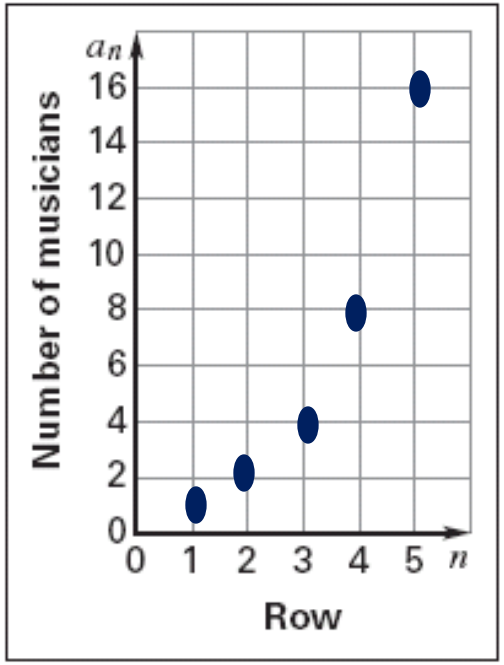
Solution

Step 1 **Make** a table showing the number of musicians in the first 3 rows. Let a_n represent the number of musicians in row n.

Row, n	1	2	3
Number of Musicians, a_n	1 $1 = 2^{1-1}$	2 $2 = 2^{2-1}$	4 $4 = 2^{3-1}$

Step 2 **Write** a rule for the number of musicians in each row. From the table, you can see that $a_n =$ 2^{n-1}.

Step 3 **Plot** the points (1, 1), (2, 2), (3, 4), (4, 8), and (5, 16). Notice that the graph is a function.



4. In Example 3, suppose the band leader wants to add a sixth row. How many musicians are needed for the sixth row?

$$a_n = 2^{n-1}$$

$$a_6 = 2^{6-1} = 2^5 = 32 \text{ musicians}$$