# 3.2 Use Special Products to Factor Cubics

Open your workbooks to page 139

*Recall…*

Multiplying polynomials.

**$(x+y)^3$** $= (x + y)(x + y)(x + y)$

$=(x + y)(x^2 + xy + xy + y^2)$

$=(x + y)(x^2 + 2xy + y^2)$

$=x^3 + 2x^2y + xy^2 + x^2y + 2xy^2 + y^3$

**$=x^3 + 3x^2y + 3xy^2 + y^3$**

$(2a+2)^3 = (2a + 2)(2a + 2)(2a + 2)$

$=(2a + 2)(4a^2 + 4a + 4a + 2^2)$

$=(2a + 2)(4a^2 + 8a + 4)$

$=8a^3 + 16a^2 + 8a + 8a^2 + 16a + 8$

**$=8a^3 + 24a^2 + 24a + 8$**

**$=(2a)^3 + 3(2a)^2 2 + 3(2a)2^2 + 2^3$**

**SPECIAL PRODUCT PATTERNS**

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

## Example 1 *Use special product patterns*

**Factor the expression.**

**a.** $x^3 + 12x^2 + 48x + 64$

**b.** $x^3 - 15x^2 + 75x - 125$

**SPECIAL PRODUCT PATTERNS**

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

**Solution**

**a.** $x^3 + 12x^2 + 48x + 64$

$= x^3 + \underline{3x^2}(\underline{4}) + \underline{3x}(\underline{4^2}) + \underline{4^3}$

$= \underline{(x + 4)^3}$

**b.** $x^3 - 15x^2 + 75x - 125$

$= x^3 - \underline{3x^2}(\underline{5}) + \underline{3x}(\underline{5^2}) - \underline{5^3}$

$= \underline{(x - 5)^3}$

**Example 2** *Factor out a monomial first*

Factor $-2x^3 + 18x^2 - 54x + 54$.

**Solution**

$-2x^3 + 18x^2 - 54x + 54$

$= -2\,(x^3 - 9x^2 + 27x - 27)$

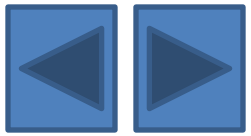
$= -2\,(x^3 - 3x^2(3) + 3x(3^2) - 3^3)$

$= -2\,(x - 3)^3$

**SPECIAL PRODUCT PATTERNS**

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$



## Example 3 *Factor cubics with multiple variables*

**Factor the expression.**

**a.** $x^3y^3 - 12x^2y^2 + 48xy - 64$

$= (xy)^3 - 3(xy)^2(4) + 3(xy)(4)^2 - 4^3$

$= (xy - 4)^3$

**b.** $x^3 + 9x^2y + 27xy^2 + 27y^3$

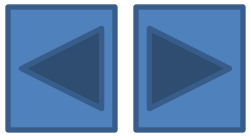
$= (x)^3 - 3x^2(3y) + 3x(3y)^2 + (3y)^3$

$= (x + 3y)^3$

**SPECIAL PRODUCT PATTERNS**

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$



**Factor the expression.**

1. $x^3 - 18x^2 + 108x - 216$

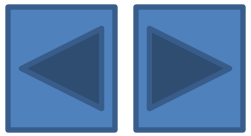
$= x^3 - 3x^2(6) + 3x(6)^2 - 6^3$

$= (x - 6)^3$

**SPECIAL PRODUCT PATTERNS**

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$



**Factor the expression.**

**2.** $8x^3 + 24x^2 + 24x + 8$

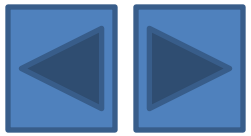
$8(x^3 + 3x^2 + 3x + 1)$

$8(x + 1)^3$

**SPECIAL PRODUCT PATTERNS**

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$



**Factor the expression.**

3. $125a^3 - 75a^2b + 15ab^2 - b^3$

$= (5a)^3 - 3(5a)^2(b) + 3(5a)(b)^2 - b^3$

$= (5a - b)^3$

**HW Book**
**pp. 132** 2 – 6 even, 7 – 12 all, 14, 16

Turn in 2, 4, 6, 8, and 10 by the end of class. The rest are homework

**SPECIAL PRODUCT PATTERNS**

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$