3.5 Solve Radical Expressions

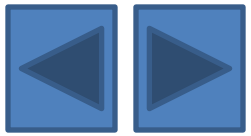
VOCABULARY

Radical equation

An equation that contains a radical expression with a variable in the radicand.

Extraneous solution

Squaring both sides of the equation $a = b$ can result in a solution of $a^2 = b^2$ that is not a solution of the original equation. Such a solution is called an extraneous solution.



Example 1 *Solve a radical equation*

Solve $3\sqrt{x+1} - 15 = -6$.

Solution

$3\sqrt{x+1} - 15 = -6$	Write original equation.
$3\sqrt{x+1} = 9$	Add 15 to each side.
$\sqrt{x+1} = 3$	Divide each side by 3.
$(\sqrt{x+1})^2 = 3^2$	Square each side.
$x + 1 = 9$	Simplify.
$x = 8$	Subtract 1 from each side.

The solution is 8.

Check the solution by substituting it in the original equation.

1. Solve $\sqrt{4x - 19} - 2 = 5$.

$$\sqrt{4x-19} = 7$$

Add 2 to both sides.

$$(\sqrt{4x-19})^2 = 7^2$$

Square both sides.

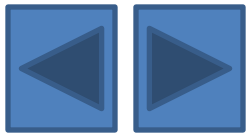
$$4x-19=49$$

$$4x=68$$

Add 19 to both sides.

$$x=17$$

Divide both sides by 4.



Solve $x = \sqrt{2x + 15}$.

Solution

$x = \sqrt{2x + 15}$ — Write original equation.

$x^2 = (\sqrt{2x+15})^2$ — Square each side.

$x^2 = 2x + 15$ — Simplify.

$x^2 - 2x - 15 = 0$ — Write in standard form.

$(x - 5)(x + 3) = 0$ — Factor.

$(x - 5) = 0$ *or* $(x + 3) = 0$

$x = 5$ *or* $x = -3$

CHECK Check 5 and –3 in the original equation.

$x =$ 5 :

$5 \stackrel{?}{=} \sqrt{2(5) + 15}$

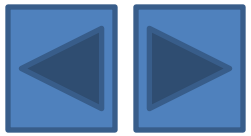
$5 = 5$ ✓

$x =$ –3 :

$-3 \stackrel{?}{=} \sqrt{2(-3) + 15}$

$-3 = 3$ ✗

Because –3 does not check in the original equation, it is an **extraneous solution**. The only solution of the equation is 5.



2. $\sqrt{30 - x} = x$

$(\sqrt{30-x})^2 = x^2$

Square both sides.

$30 - x = x^2$

$-x^2 - x + 30 = 0$

Write in standard form.

$-(x^2 + x - 30) = 0$

Factor out a –1 .

$-(x + 6)(x - 5) = 0$

Factor.

$x + 6 = 0$ $\quad x - 5 = 0$

$x = -6$ $\quad x = 5$

$\sqrt{30-(-6)} = -6$

$\sqrt{36} = -6$

$6 \neq -6$

$\sqrt{30-(5)} = 5$

$\sqrt{25} = 5$

$5 = 5$ solution

3. $\sqrt{7 + 6x} = x$

$(\sqrt{7+6x})^2 = x^2$

Square both sides.

$7 + 6x = x^2$

$-x^2 + 6x + 7 = 0$

Write in standard form.

$-(x^2 - 6x - 7) = 0$

Factor out a –1 .

$-(x + 1)(x - 7) = 0$

Factor.

$x + 1 = 0$ $\quad x - 7 = 0$

$x = -1$ $\quad x = 7$

$\sqrt{7+6(-1)} = -1$

$\sqrt{7-6} = -6$

$\sqrt{1} = -1$

$1 \neq -1$

$\sqrt{7+6(7)} = 7$

$\sqrt{7+42} = 7$

$\sqrt{49} = 7$

$7 = 7$ solution

Your Notes

Example 3 *Solve an equation with radicals on both sides*

Solve $\sqrt{3x - 3} = \sqrt{2x + 8}$.

Solution

$\sqrt{3x - 3} = \sqrt{2x + 8}$	Write original equation.
$(\sqrt{3x-3})^2 = (\sqrt{2x+8})^2$	Square each side.
$3x-3 = 2x+8$	Simplify.
$x-3 = 8$	Subtract $2x$ from each side.
$x = 11$	Add 3 to each side.

The solution is 11. Check the solution.

To solve a radical equation that contains two radical expressions, be sure that each side of the equation has only one radical expression before squaring each side.

4. $\sqrt{5x - 4} = \sqrt{3x + 20}$

$(\sqrt{5x-4})^2 = (\sqrt{3x+20})^2$ **Square both sides.**

$5x-4=3x+20$ **Simplify.**

$2x-4=20$ **Subtract 3x from both sides.**

$2x=24$ **Add 4 to both sides.**

$x=12$ **Divide both sides by 2.**

Homework:
Textbook
p. 148 2 – 30 even

5. $\sqrt{13 - x} = \sqrt{3x - 15}$

$$(\sqrt{13-x})^2 = (\sqrt{3x-15})^2$$ **Square both sides.**

$$13 - x = 3x - 15$$ **Simplify.**

$$13 = 4x - 15$$ **Add x to both sides.**

$$28 = 4x$$ **Add 15 to both sides.**

$$x = 7$$ **Divide both sides by 4.**