

3.8 Simplify Rational Expressions

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Rational expression

An expression that can be written as the ratio of two polynomials.

Excluded value

A number that makes a rational expression undefined.

Simplest form of a rational expression

A rational expression is in simplest form if the numerator and denominator have no factors in common other than 1 .



Example 1 Find excluded values

Find the excluded values, if any, of the expression.

a. $\frac{x}{4x - 8}$

b. $\frac{3x}{x^2 - 16}$

Solution

a. The expression $\frac{x}{4x - 8}$ is undefined when $4x - 8$ = 0, or $x =$ 2. The excluded value is 2.

b. The expression $\frac{3x}{x^2 - 16}$ is undefined when $x^2 - 16$ = 0, or $($ $x + 4$ $)$ $($ $x - 4$ $) = 0$.

The solutions of the equation are -4 and 4.

The excluded values are -4 and 4.



Example 1 Find excluded values

Find the excluded values, if any, of the expression.

c. $\frac{1}{x^2 + 2}$

c. The expression $\frac{1}{x^2 + 2}$ is undefined when $x^2 + 2 = 0$.

The graph of $y = x^2 + 2$ does not cross the x-axis.

So, the quadratic equation has no real roots. There are no excluded values.



$$1. \frac{x + 6}{14x}$$

$$14x = 0$$

$$x = 0$$

The excluded value is 0.

$$2. \frac{9x + 1}{x^2 - x - 20}$$

$$x^2 - x - 20 = 0$$

$$(x - 5)(x + 4) = 0$$

$$x - 5 = 0$$

$$x + 4 = 0$$

$$x = 5$$

$$x = -4$$

The excluded values are -4 and 5 .



Example 2**Simplify expressions by dividing out monomials**

Simplify the rational expression, if possible. State the excluded values.

$$\begin{aligned} \text{a. } \frac{18x}{6x^2} &= \frac{\cancel{6} \cdot 3 \cdot \cancel{x}}{\cancel{6} \cdot \cancel{x} \cdot x} \\ &= \frac{3}{x} \end{aligned}$$

Divide out common factors.

Simplify.

The excluded value is 0.

$$\begin{aligned} \text{b. } \frac{12x^2 - 6x}{24x} &= \frac{6 \cdot x \cdot (2x - 1)}{6 \cdot 4 \cdot x} \\ &= \frac{\cancel{6} \cdot \cancel{x} \cdot (2x - 1)}{\cancel{6} \cdot 4 \cdot \cancel{x}} \\ &= \frac{2x - 1}{4} \end{aligned}$$

Factor numerator and denominator.

Divide out common factors.

Simplify.

The excluded value is 0.



$$3. \frac{7}{5x + 3}$$

$$5x + 3 = 0$$

$$5x = -3$$

$$x = -\frac{3}{5}$$

$$4. \frac{5x}{5x^2 - 25}$$

$$\frac{\cancel{5} \cdot x}{\cancel{5}(x^2 - 5)}$$

$$\frac{x}{x^2 - 5}$$

$$x^2 - 5 = 0$$

$$x^2 = 5$$

$$x = \pm\sqrt{5}$$

$$5. \frac{6x^3}{2x + 4}$$

$$\frac{\cancel{2} \cdot 3 \cdot x \cdot x \cdot x}{\cancel{2}(x + 2)}$$

$$\frac{3x^3}{(x + 2)}$$

$$x + 2 = 0$$

$$x = -2$$



Example 3*Simplify an expression by dividing out binomials*

Simplify $\frac{x^2 + x - 12}{x^2 - 5x + 6}$. State the excluded values.

Solution

$$\begin{aligned}\frac{x^2 + x - 12}{x^2 - 5x + 6} &= \frac{(x - 3)(x + 4)}{(x - 3)(x - 2)} \\ &= \frac{(x + 4)}{(x - 2)}\end{aligned}$$

Factor and divide
out common factor.

Simplify.

The excluded values are 3 and 2.



Checkpoint Simplify the rational expression. State the excluded values.

6. $\frac{x^2 + 7x + 6}{x^2 + 3x - 18}$

$$\frac{(x + 6)(x + 1)}{(x + 6)(x - 3)}$$

$$\frac{(x + 1)}{(x - 3)}$$

$$x - 3 = 0$$

$$x = 3$$

$$x + 6 = 0$$

$$x = -6$$



Checkpoint Simplify the rational expression. State the excluded values.

$$7. \frac{-(x^2 - 4)}{x^2 + 5x - 14} = \frac{-(x + 2)(x - 2)}{(x + 7)(x - 2)}$$

$$\frac{-(x + 2)}{(x + 7)}$$

$$x - 2 = 0$$

$$x + 7 = 0$$

$$x = 2$$

$$x = -7$$

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Due FRIDAY 4/3/09 Tomorrow!!!!!!

