

3.9 Multiply and Divide Rational Expressions



Example 1**Multiply rational expressions involving polynomials**

Find the product $\frac{x}{5x^2 - 6x - 8} \cdot \frac{2x^2 - 4x}{7x^2}$.

$$\frac{x}{5x^2 - 6x - 8} \cdot \frac{2x^2 - 4x}{7x^2}$$

$$= \frac{x(2x^2 - 4x)}{(5x^2 - 6x - 8)(7x^2)}$$

$$= \frac{2x^2(x-2)}{7x^2(5x+4)(x-2)}$$

$$= \frac{2}{7(5x+4)}$$

Multiply numerators and denominators.

Factor and divide out common factors.

Simplify.



Example 2**Multiply a rational expression by a polynomial**

Find the product $\frac{4x}{x^2 - x - 12} \cdot (x - 4)$.

$$\frac{4x}{x^2 - x - 12} \cdot (x - 4)$$

$$= \frac{4x}{x^2 - x - 12} \cdot \frac{x - 4}{1}$$

Rewrite polynomial as a fraction.

$$= \frac{4x(x - 4)}{x^2 - x - 12}$$

Multiply numerators and denominators.

$$= \frac{4x\cancel{(x - 4)}}{(x + 3)\cancel{(x - 4)}}$$

Factor and divide out common factor.

$$= \frac{4x}{x + 3}$$

Simplify.



$$1. \frac{x^2 - 5x + 4}{3x^2 - 12x} \cdot \frac{2x^2 + 2}{x^2 + 6x - 7} = \frac{(x^2 - 5x + 4)(2x^2 + 2)}{(3x^2 - 12x)(x^2 + 6x - 7)}$$

$$= \frac{\cancel{(x-4)}\cancel{(x-1)}(2)(x^2 + 1)}{3x\cancel{(x-4)}(x+7)\cancel{(x-1)}} = \frac{2(x^2 + 1)}{3x(x+7)}$$



$$2. \frac{2x}{x^2 + 5x - 24} \cdot (x + 8)$$

$$= \frac{(2x)(x + 8)}{x^2 + 5x - 24}$$

$$= \frac{(2x)\cancel{(x + 8)}}{(x - 3)\cancel{(x + 8)}}$$

$$= \frac{2x}{x - 3}$$



Example 3**Divide rational expressions involving polynomials**

Find the quotient $\frac{x^2 + 5x - 24}{x^2 + 9x + 8} \div \frac{x^2 - 9}{6x - 18}$.

Solution

$$\frac{x^2 + 5x - 24}{x^2 + 9x + 8} \div \frac{x^2 - 9}{6x - 18}$$

$$= \frac{x^2 + 5x - 24}{x^2 + 9x + 8} \cdot \frac{6x - 18}{x^2 - 9}$$

Multiply by
multiplicative inverse.

$$= \frac{(x^2 + 5x - 24)(6x - 18)}{(x^2 + 9x + 8)(x^2 - 9)}$$

Multiply numerators
and denominators.

$$= \frac{(x - 3)(x + 8)(6)(x - 3)}{(x + 1)(x + 8)(x + 3)(x - 3)}$$

Factor and divide out
common factors.

$$= \frac{2}{7(5x + 4)}$$

Simplify.



Example 4**Divide a rational expression by a polynomial**

Find the quotient $\frac{x^2 - 25}{x - 3} \div (x - 5)$.

Solution

$$\frac{x^2 - 25}{x - 3} \div (x - 5)$$

$$= \frac{x^2 - 25}{x - 3} \div \frac{x - 5}{1}$$

Rewrite polynomial as fraction.

$$= \frac{x^2 - 25}{x - 3} \cdot \frac{1}{x - 5}$$

Multiply by multiplicative inverse.

$$= \frac{x^2 - 25}{(x - 3)(x - 5)}$$

Multiply numerators and denominators.

$$= \frac{(x + 5)\cancel{(x - 5)}}{(x - 3)\cancel{(x - 5)}}$$

Factor and divide out common factor.

$$= \frac{x + 5}{x - 3}$$

Simplify.



$$\begin{aligned} 3. \quad \frac{x^2 + 2x - 15}{x^2 + 4x - 5} \div \frac{x^2 - 4}{7x - 14} &= \frac{x^2 + 2x - 15}{x^2 + 4x - 5} \bullet \frac{7x - 14}{x^2 - 4} \\ &= \frac{\cancel{(x+5)}(x-3)(7)\cancel{(x-2)}}{\cancel{(x+5)}(x-1)(x+2)\cancel{(x-2)}} \\ &= \frac{7(x-3)}{(x-1)(x+2)} \end{aligned}$$



$$4. \frac{x^2 + 8x + 7}{x^2 - 1} \div (x + 7) = \frac{x^2 + 8x + 7}{x^2 - 1} \bullet \frac{1}{x + 7}$$

$$= \frac{\cancel{(x + 7)}\cancel{(x + 1)}}{\cancel{(x + 1)}(x - 1)\cancel{(x + 7)}}$$

$$= \frac{1}{x - 1}$$

