4.2 Use Inductive Reasoning

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VOCABULARY

Conjecture

An unproven statement that is based on observations.

Inductive Reasoning

The process of finding a pattern for specific cases and then writing a conjecture for the general case.

Counterexample

A specific case for which a conjecture is false.

Example 1 *Describe a visual pattern*

Describe how to sketch the fourth figure in the pattern. Then sketch the fourth figure.

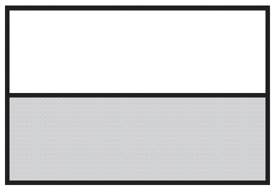
Figure 1


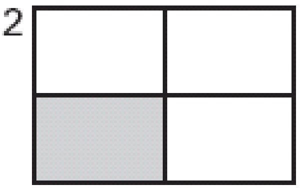
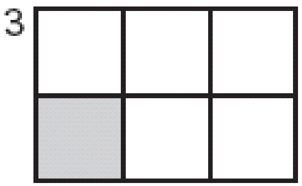
Figure 2


Figure 3


Solution

Each rectangle is divided into ________ as many equal regions as the figure number. Sketch the fourth figure by dividing the rectangle into ________. Shade the section just ________ the horizontal segment at the _____.

____

Figure 4

1. Sketch the fifth figure in the pattern in Example 1.

Figure 1

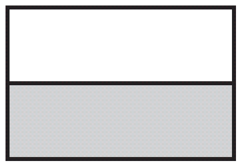
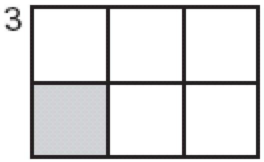


Figure 2

Figure 3



Example 2 ***Describe a number pattern***

Describe the pattern in the numbers $-1, -4, -16, -64, \ldots$ and write the next three numbers in the pattern.

Notice that each number in the pattern is ______ times the previous number.

$-1, \quad -4, \quad -16, \quad -64, \ldots$

$\times$ ___ $\quad \times$ ___ $\quad \times$ ___ $\quad \times$ ___

The next three numbers are ______________________.

2. Describe the pattern in the numbers 1, 2.5, 4, 5.5, . . . and write the next three numbers in the pattern.

Example 3 *Make and test a conjecture*

Numbers such as 1, 3, and 5 are called consecutive odd numbers. Make and test a conjecture about the sum of any three consecutive odd numbers.

Step 1 **Find** a pattern using groups of small numbers.

$1 + 3 + 5 = \_\_\_$	$3 + 5 + 7 = \_\_\_$
$= 3 \cdot 3$	$= \underline{5} \cdot 3$
$5 + 7 + 9 = \_\_\_$	$7 + 9 + 11 = \_\_\_$
$= \_\_\_ \cdot 3$	$= \_\_\_ \cdot 3$

Conjecture The sum of any three consecutive odd numbers is three times ____________________.

Step 2 **Test** your conjecture using other numbers.

$-1 + 1 + 3 = \_\_\_ = \_\_\_ \cdot 3$ ✓

$103 + 105 + 107 = \_\_\_ = \_\_\_ \cdot 3$ ✓

3. Make and test a conjecture about the sign of the product of any four negative numbers.

Example 4 *Find a counterexample*

A student makes the following conjecture about the difference of two numbers. Find a counterexample to disprove the student's conjecture.

Conjecture The difference of any two numbers is always smaller than the larger number.

Solution

To find a counterexample, you need to find a difference that is __________ than the __________ number.

$8 - (-4) =$ _____

Because _____ $\not<$ ___, a counterexample exists. The conjecture is false.

4. Find a counterexample to show that the following conjecture is false.

Conjecture The quotient of two numbers is always smaller than the dividend.

Textbook

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