Algebra III

(Advanced Mathematics)

Lesson 1

Geometry Review

Basic Terms

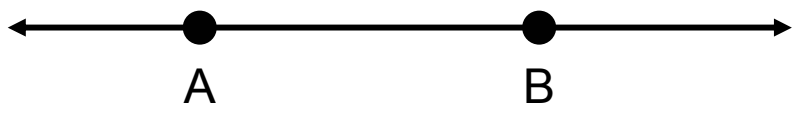
Point - **Has no size, but is a location. A spot on a paper isn't a point, but it is a marker for a point**

Curve - **The path of a moving point, or a series of connected points**

Examples:

Lines - **A straight curve. Connects at 2 points and never ends**

Example:



Written as $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$

Segment - A part of a line, includes the end points and everything in between.

Written as $\overline{AB}$ or $\overline{BA}$

AB or BA (without the bar on top) means the length of $\overline{AB}$ or $\overline{BA}$

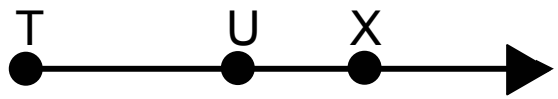
Note: <, >, or = compare measures or numbers , not shapes or geometric figures.

Measurements are equal (=); Geometric figures are congruent (≅).

DE = GH

$\overline{DE} \cong \overline{GH}$

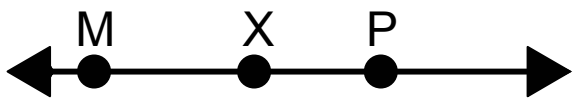
Ray - (a half line) starts at a point goes to and through another point indefinitely



T is the origin of the ray

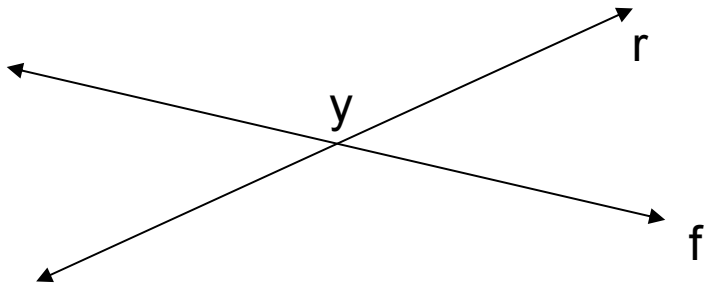
$\overrightarrow{TU}$, $\overrightarrow{TX}$ not $\overrightarrow{XT}$

Opposite Rays - Two rays that point in opposite directions and are collinear



$\overrightarrow{XM}$ and $\overrightarrow{XP}$ are collinear

Intersection - Where two figures have points in common.



Line r intersects line f at point y

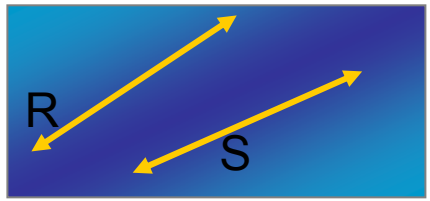
Plane - Has no thickness and extends indefinitely in two directions

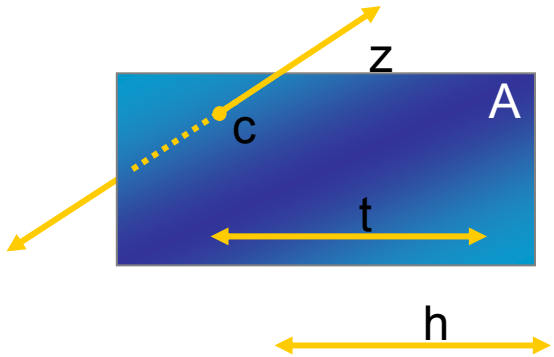
A table top that has no end

Plane P

Plane Q

Coplanar - Geometric figures that exist in the same plane.





$\overleftrightarrow{R}$ is coplanar with $\overleftrightarrow{S}$

$\overleftrightarrow{h}$ does not intersect Plane A

$\overleftrightarrow{t}$ is parallel (//) to $\overleftrightarrow{h}$

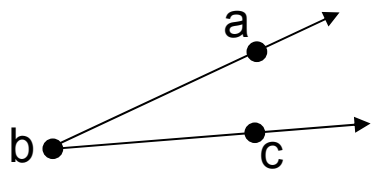
Since $\overleftrightarrow{z}$ intersects plane A at point c, $\overleftrightarrow{z}$ is a skew line.

$\overleftrightarrow{h}$ is not a skew line to plane A since they don't intersect

Angles -

def. #1

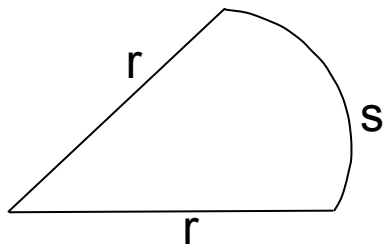
Two rays having a common origin form the geometric figure of an angle. The opening between the two rays can be measured to find the measure of the angle.



Angles -

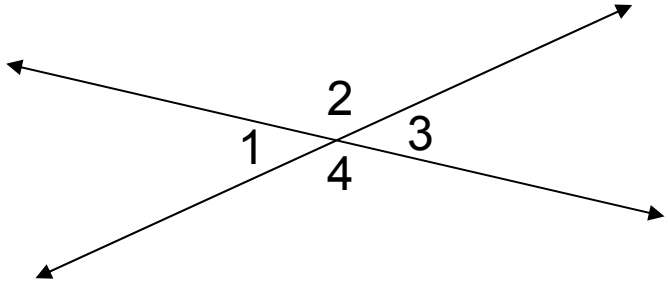
def. #2

The region bounded by two radii and the arc of a circle. The measure of the angle is the ratio of the length of the arc to the length of the radius.

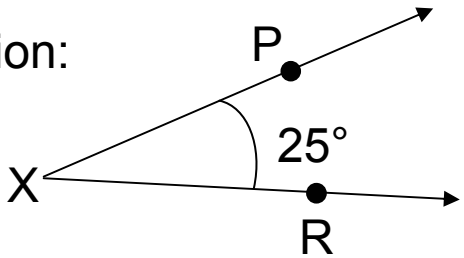


Angles - The difference in direction of two intersecting lines.

def. #3



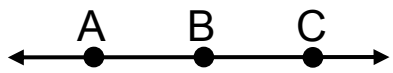
Notation:



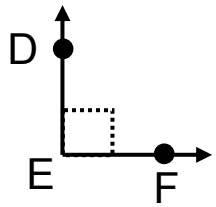
$\angle X$
$\angle PXR$
$\angle RXP$

Same thing

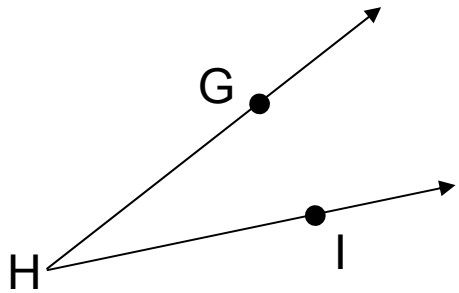
Straight Angle ⟶ 180°



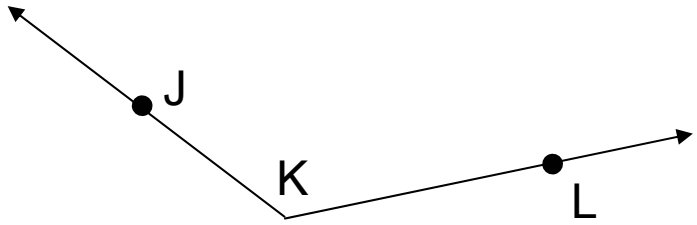
Right Angle ⟶ 90°
(perpendicular)



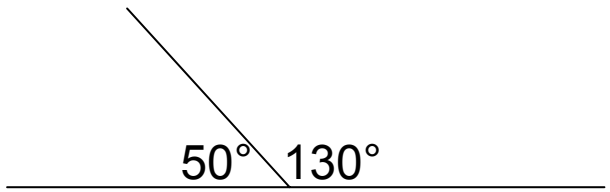
Acute Angle between 0 and 90°

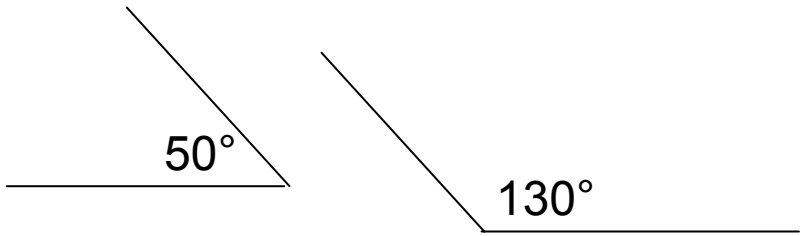


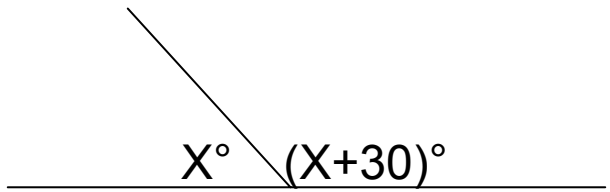
Obtuse Angles between 90° and 180°



Supplementary Angles – The measure of two angles adding to 180°







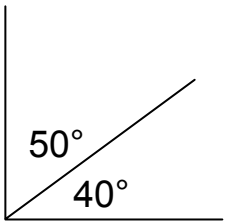
$X + (X+30) = 180°$

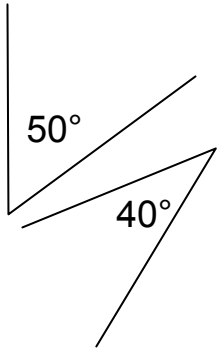
$2X+30=180°$

$2X = 150°$

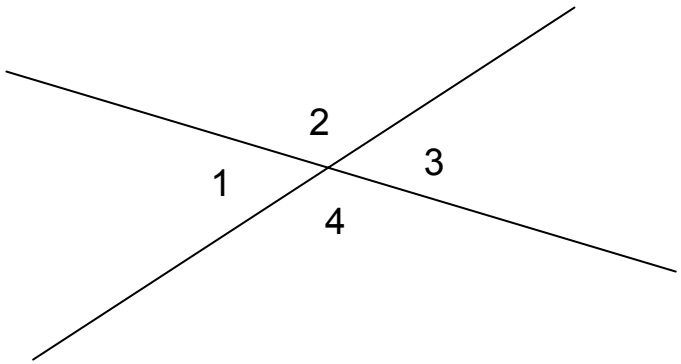
$X = 75°$

Complementary Angles - The measure of two angles equals 90°





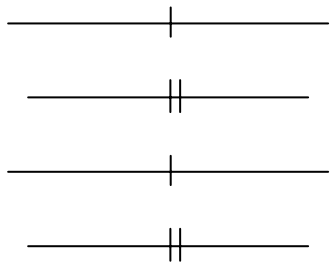
Vertical Angles - Opposite wedges from intersecting lines are always equal.

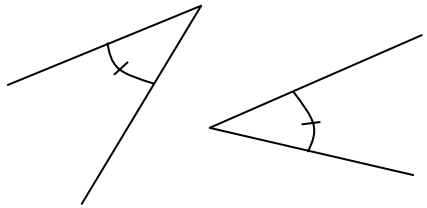


1 & 3 and 2 & 4 are vertical angles.

Betweenness - At least three collinear points, where the one(s) in the middle are between the others.

Tick Marks - Show the properties of figures that are the same.





Assumption Do's and Don'ts

- Do assume straight lines
- Do not assume perpendicular
- Do not assume parallel
- Do not assume equality of measures
- Do assume collinear points

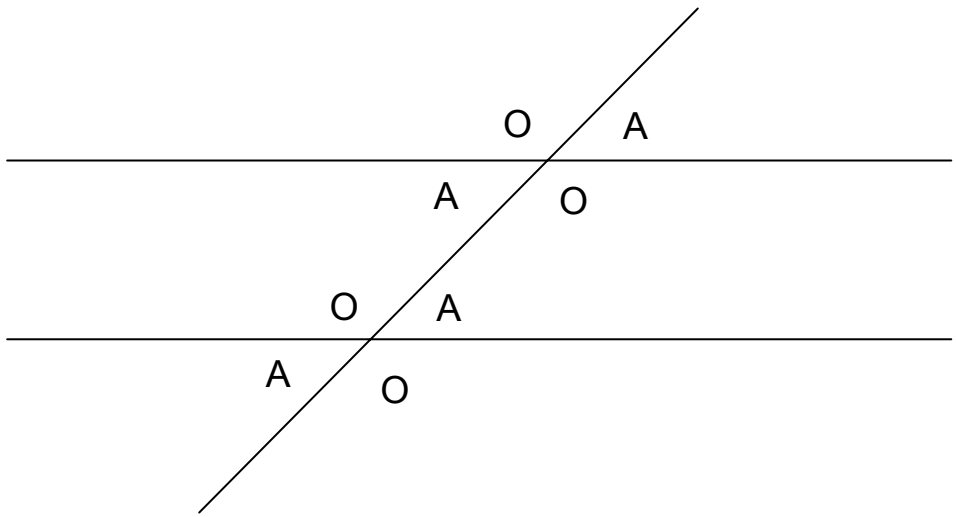
Triangles -

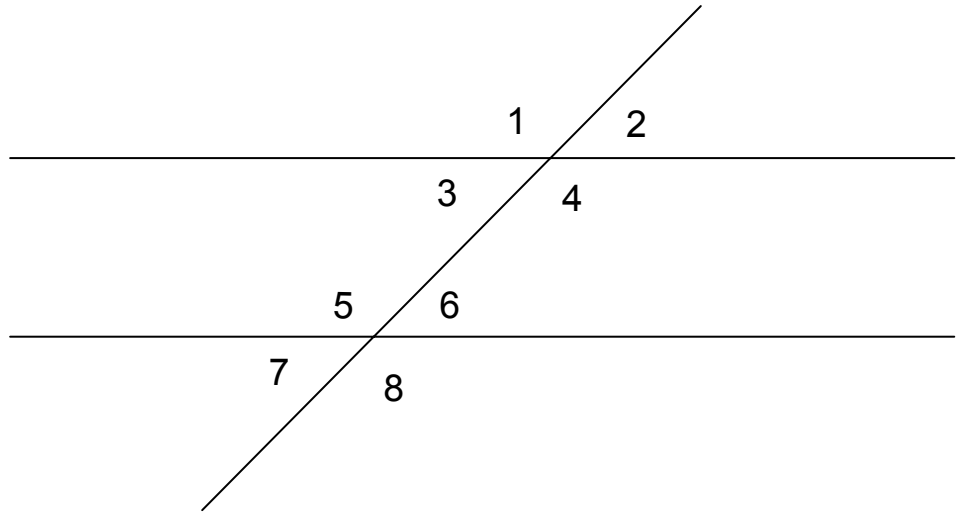
Equiangular -	Three equal angles
Isogonic -	At least two equal angles
Right -	Has a 90°
Acute -	All angles less than 90°
Obtuse -	One angle between 90° and 180°
Equilateral -	Three equal sides
Isosceles -	At least two equal sides
Scalene -	No equal sides

Parallel Line Topics

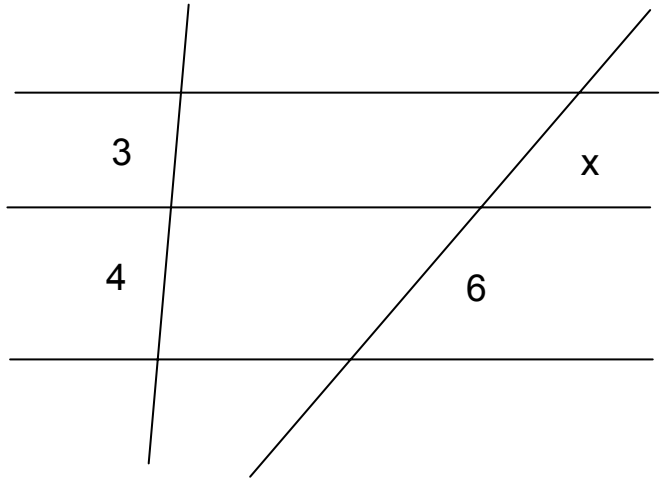
Transversal - A line cutting across a set of parallel lines

When a transversal cuts parallel lines two sets of equal measure angles: one set is acute, and one set is obtuse.





Exterior Angles -	1, 2, 7, 8 – to the outside of the parallel lines
Interior Angles -	3, 4, 5, 6 – between the parallel lines
Alternate Angles -	Opposite sides of the transversal – 1 & (2, 4, 6, 8)
Corresponding Angles -	Angles with the same relative position on each of the parallel lines – 1 & 5, 2 & 6, 3 & 7, 4 & 8
Alternate Interior Angles -	3 & 6 and 5 & 4 – always equal
Alternate Exterior Angles -	1 & 8 and 2 & 7 – always equal



$$\frac{3}{4} = \frac{x}{6}$$

$$4x = 18$$

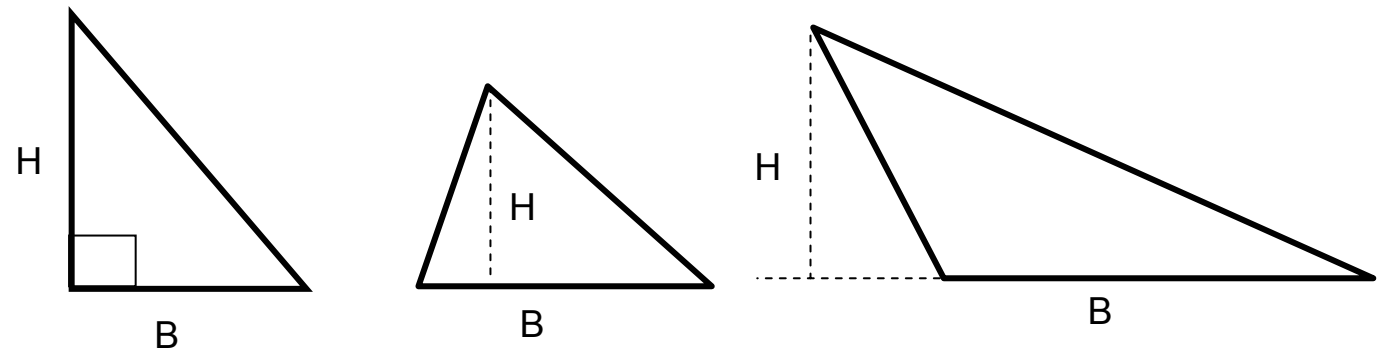
$$x = \frac{18}{4}$$

$$x = \frac{9}{2}$$

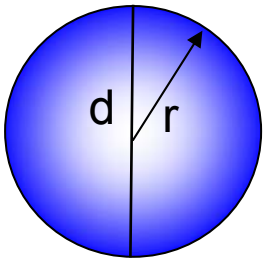
Leave the answer this way. Not as 4.5.

Areas & Sectors of Circles

$A = L * W$



$A = \frac{1}{2} BH$

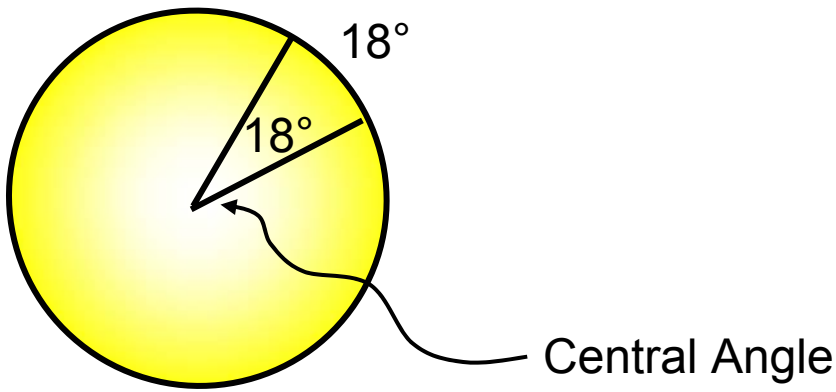


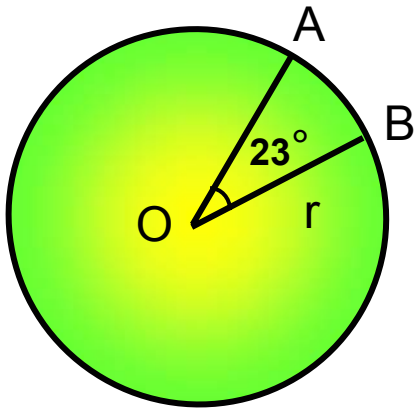
$A = \pi r^2$

$C = \pi d = 2\pi r$ (use $2\pi r$)

Arc - A piece of the circle.

Sector - A wedge of sector.



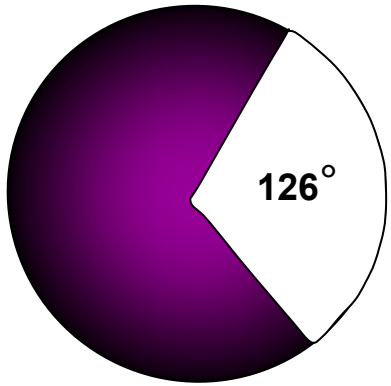


Area of Sector AOB = part of Area of Circle

$$A_{aob} = \frac{23}{360} \times \pi r^2$$

Example 1.1

The measure of the central angle of the un-shaded sector is 126°. The radius of the circle is $2\sqrt{5}$ m. Find the area of the shaded region of the circle



$$A_{shade} = \frac{234}{360}\pi\left(2\sqrt{5}\right)^2$$

$$= \frac{117}{180}\pi\left(4 \bullet 5\right)$$

$$= \frac{117}{180}\pi\left(20\right)$$

$$= \frac{117}{9}\pi$$

$$= 13\ \pi m^2$$

(leave answer this way)

Concept Review Problems

These will compare two concepts. A & B

The answers are:

A if A is greater

B if B is greater

C if A = B

D if can't tell

Can be solved by using subtraction.

A - B

If A-B>0 then A is bigger

If A-B<0 then B is bigger

If A-B=0 then C is the answer

Otherwise D is the correct answer

Example 1.2

Let X and Y be real numbers. If X>0 and Y<0

Compare: A. X+Y B. X-Y

$A-B = (X+Y) - (X-Y)$

$= X + Y - X + Y$

$= 2Y$

Since $Y<0$, $2Y<0$

So $A-B<0$

$(X+Y) - (X-Y) < 0$

$+(X-Y) \quad + (X-Y)$

$(x + Y) < (X - Y)$

$A < B$ So B is the answer

Example 1.3

Let X AND Y be real numbers. If $1 < X < 6$ and $1 < Y < 6$,

Compare: A. X-Y B. Y-X

Three possible scenarios. Since X & Y have ranges

1) $X<Y$ then X-Y is negative and Y-X is positive, making B the answer

2) $X>Y$ then X-Y is positive and Y-X is negative, so A is the answer

3) $X=Y$ then $X-Y=0$ and $Y-X=0$ so C is the answer

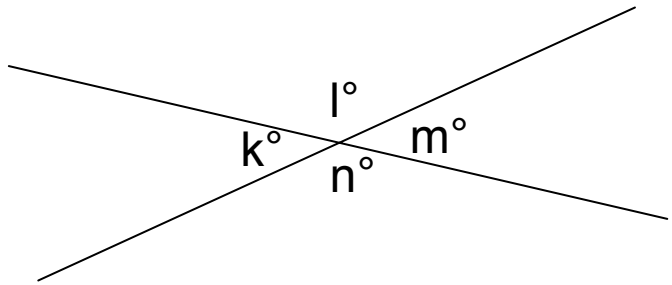
Therefore D is the real answer

Example 1.4

Given two intersecting lines with angles k. l, m, and n, as shown

Compare: A. (k + l + m)° B. (180 – n)°

Note: k+l = 180°



So A becomes 180 + m

Note: m+n = 180

So B becomes 180-n=m

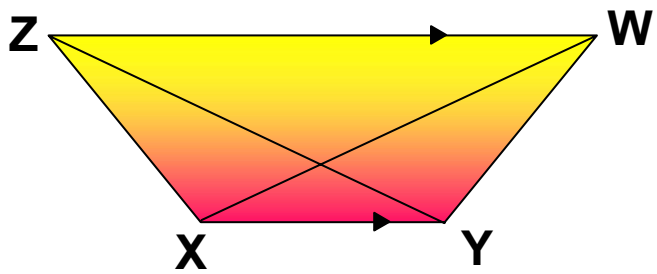
A. 180+m B. m

A > B

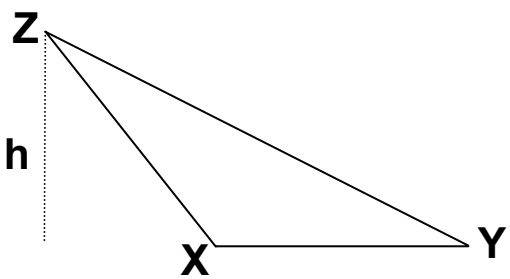
Example 1.5

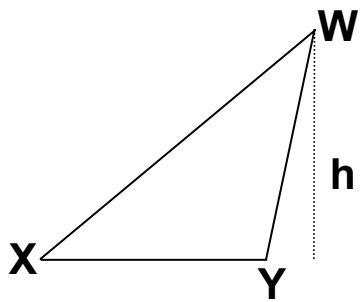
Given ZW and XY are parallel, as shown:

Compare: A. Area of △WXY° B. Area of △ZXY



Find area of both △s and compare





$A = \frac{1}{2} bh$

$= \frac{1}{2} (xy)h$

$A = \frac{1}{2} bh$

$= \frac{1}{2} (xy)h$

So A=B

Therefore C is the answer

Practice

A) Find the measure of angle A if five times the complement of angle A is 150 greater than the supplement of A. Remember the complement of A is 90 – A and the supplement of A is 180 – A.

$$5(90 - A) = 150 + (180 - A)$$

$$450 - 5A = 330 - A$$

$$450 - 330 = 5A - A$$

$$120 = 4A$$

$$30 = A$$

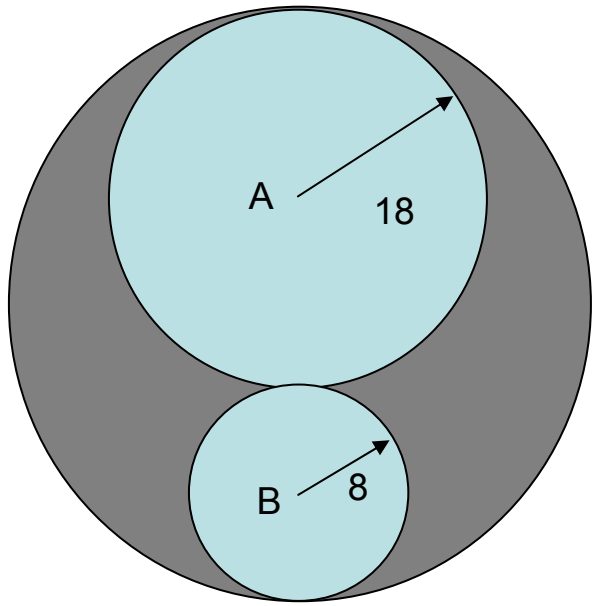
B) Simplify $\dfrac{2^{-3}x^{0}(x^{2})}{x^{-3}xy^{-3}y}$

$$= \frac{x^{2}}{8x^{-2}y^{-2}}$$

$$= \frac{x^{4}y^{2}}{8}$$

C) In this figure, points A and B are the centers of two smaller circles and lie on a diameter of the big circle. The two smaller circles are tangent to the larger circle and to each other. Find the area of the shaded region of this figure. Dimensions are in centimeters.

Start with the big picture and remove the unwanted parts.



$A_{shaded} = A_{big} - A_{middle} - A_{small}$

$A_{big} = \pi r_b^{\,2}$ $\qquad$ $A_{middle} = \pi r_m^{\,2}$ $\qquad$ $A_{small} = \pi r_s^{\,2}$

$r_b = \dfrac{(18+18+8+8)}{2}$ $\qquad$ $A_{middle} = \pi(18)^2$ $\qquad$ $A_{small} = \pi(8)^2$

$r_b = \dfrac{52}{2} = 26$ $\qquad$ $A_{middle} = 324\pi$ $\qquad$ $A_{small} = 64\pi$

$A_{big} = \pi(26)^2$

$A_{big} = 676\pi$

$A_{shaded} = 676\pi - 324\pi - 64\pi$

$A_{shaded} = 288\pi \text{ cm}^2$

D) Compare: A. $\sqrt{\frac{1}{4}}+\sqrt{\frac{1}{25}}$ B. $\sqrt{\frac{1}{4}+\frac{1}{25}}$

Simplify each piece then compare

A. $\sqrt{\frac{1}{4}}+\sqrt{\frac{1}{25}}$

$=\frac{1}{2}+\frac{1}{5}$

$=\frac{5}{10}+\frac{2}{10}$

$=\frac{7}{10}$

B. $\sqrt{\frac{1}{4}+\frac{1}{25}}$

$=\sqrt{\frac{25}{100}+\frac{4}{100}}$

$=\sqrt{\frac{29}{100}}$

$=\frac{\sqrt{29}}{10}$

Since the square root of 29 is between 5 and 6, A is bigger and the answer.