Algebra III

Lesson 2

More on Area ~ Cylinders and Prisms ~ Cones and Pyramids ~ Spheres

More on Area

Area is measured in square units. Area is a 2-D concept.

For example: cm^2, m^2, in^2, mi^2

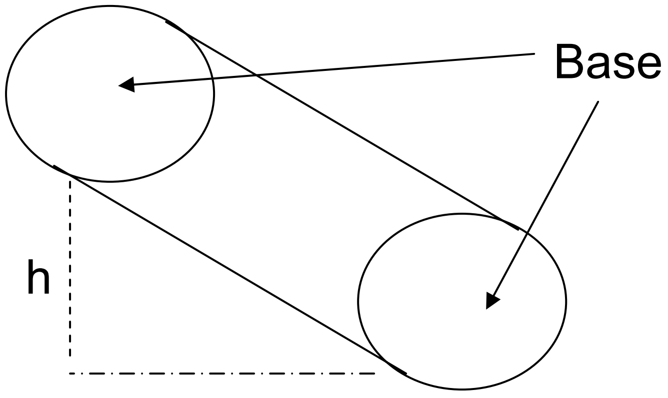
More on Cylinders and Prisms

Cylinders and prisms are the same thing - - tubes. They only differ in the cross-section shape of the tube (circular, triangular, rectangular).

Volume $V = (A_{Base})(h)$

The base is the part of the tube it would *stand up* on.

Volume is measured in cubic units (cm^3, in^3). Volume is a 3-D concept.



Lateral Surface Area (LSA)

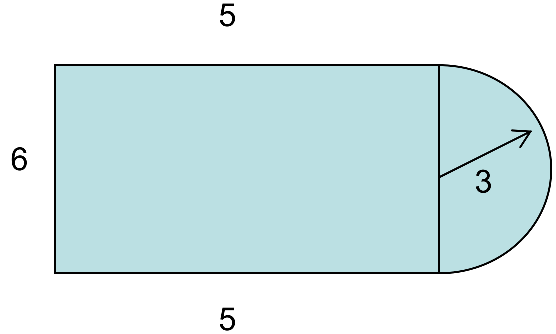
This is the total area of the sides only, not the top or bottom.

LSA = (perimeter)(height)

Note: unroll the sides of any shape cylinder and you have a rectangle

Example 2.1

The base of a right cylinder 10 centimeters high is shown. Find the surface area of the right cylinder. Dimensions are in centimeters.



$A_{Base} = A_{rect} + ½(A_{circ})$

$= (lw) + ½(\pi r^2)$

$= 30 + \frac{9}{2}\pi \text{ cm}^2$

Surface Area = LSA + 2($Area_{Base}$)

LSA = (perimeter)(height)

Perimeter = 5 + 5 + 6 +1/2(2πr)

= 16 + 3π

LSA = (16 + 3π)(10)

$= 160 + 30\pi \text{ cm}^2$

$$\text{Surface Area} = (160 + 30\pi) + 2\left(30 + \frac{9}{2}\pi\right)$$

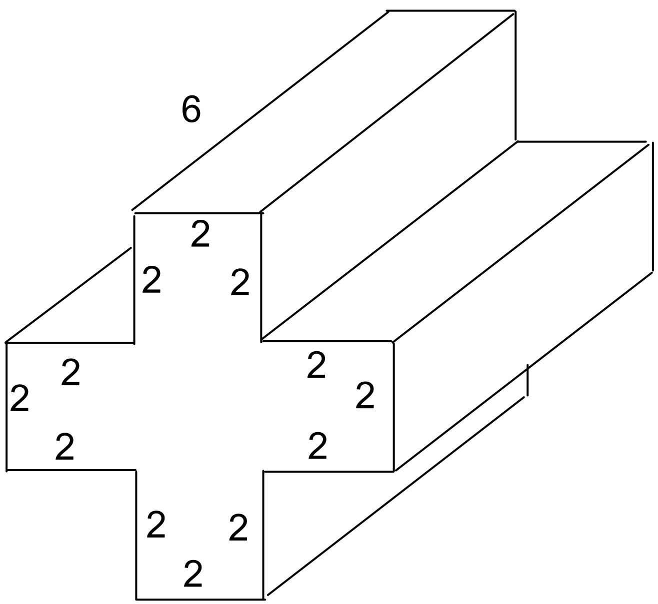
= 160 + 30π + 60 + 9π

$= 220 + 39\pi \text{ cm}^2$

$= 342.53 \text{ cm}^2$

Example 2.2

Find the surface area of this right prism.



Surface Area = LSA + 2(A_{base})

LSA = (perimeter)(height)

= (12 * 2)(6)

= 24 * 6

= 144

A_{base} = 5(2 * 2)

= 20

Surface Area = 144 + 2(20)

= 184 cm^2

Cones and Pyramids

These shapes come to a point. And are the same thing, except for the base.

Volume

Volume = 1/3 of the volume of the with the same base and height.

$= 1/3\ (A_{base})(\text{Height})$

Surface Area

Surface Area (SA) = LSA + A_{base}

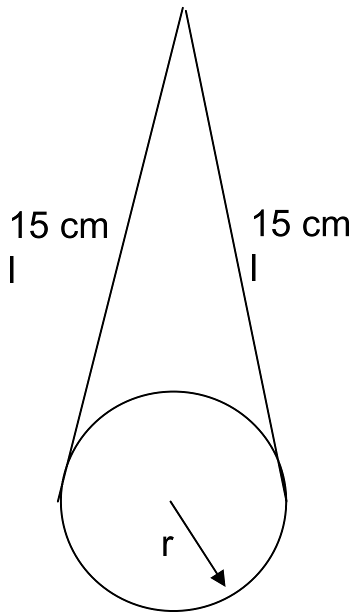
LSA of a Circular Cone

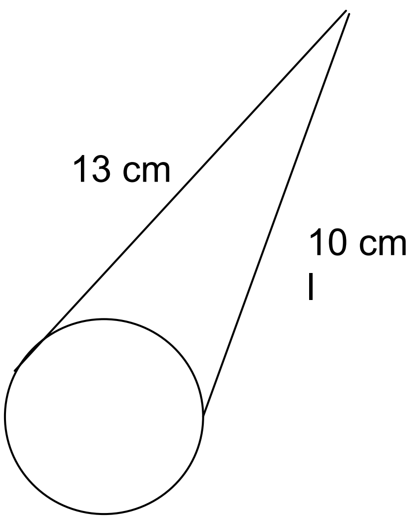
LSA = πrl

r → radius of the base

l → slant height

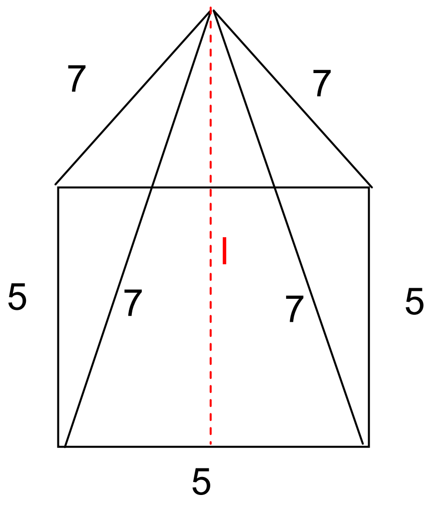
Slant height is the shortest distance from the tip of any cone to its base.





Lateral Surface Area of a Pyramid

The lateral surface area of a pyramid has no formula. It is found by adding together the areas of each side individually. Each side should be a triangle.



The slant height, l, is the height of the triangular side of the pyramid, as long as the pyramid is not flattened sideways. If it is pushed sideways at the tip just treat each side different.

Example 2.4

The radius of the base of a right circular cone is 8 cm. The slant height of the cone is 10 cm. Find the surface area of the cone.

$$SA = LSA + A_{base}$$

$$LSA = \pi r l \qquad\qquad A_{base} = \pi r^2$$

$$= \pi(8)(10) \qquad\qquad = \pi(8)^2$$

$$= 80\pi \text{ cm}^2 \qquad\qquad = 64\pi \text{ cm}^2$$

$$SA = 80\pi + 64\pi$$

$$= 144\pi \text{ cm}^2$$

Example 2.5

The volume of a right circular cone is 6π cm^3. The height of this right circular cone is 2 cm. Find the radius of the base of the cone.

The radius of the cone is used to find the Area$_{base}$. So, use the volume to find the area to find the radius.

$$V = \frac{1}{3} A_{base} h$$

$$A_{base} = \frac{3V}{h}$$

$$= \frac{3 \bullet 6\pi}{2}$$

$= 9\pi$ cm^2

$A_{base} = \pi r^2$

$$r = \sqrt{\frac{A_{base}}{\pi}}$$

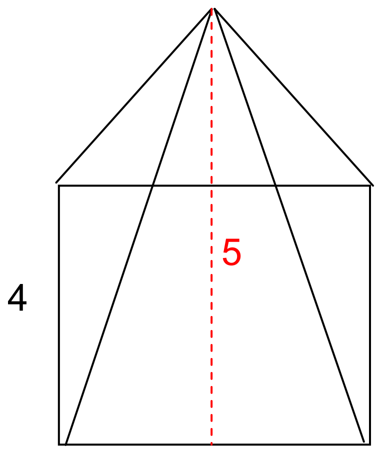
$$r = \sqrt{\frac{9\pi}{\pi}}$$

Note: the π's cancelled out.

The radius is 3 cm.

Example 2.6

The base of a regular pyramid is a square with each side measuring 4 cm. The slant height of the pyramid is 5 cm. Find the surface of the pyramid.



$SA = LSA + A_{base}$

$LSA = 4(A_{side})$

$= 4(½bh)$

$= 2bh$

$= 2(4)(5)$

$= 40\ cm^2$

$A_{base} = lw$

$= (4)(4)$

$= 16\ cm^2$

$SA = 40 + 16$

$= 56\ cm^2$

Spheres

Surface Area

SA = $4\pi r^2$

Volume

V = $\frac{4}{3}\pi r^3$

Another way to find the volume

V = 2/3 of the volume of the smallest cylinder that can hold the sphere.

V = 2/3 $(A_{base})(h)$

V = 2/3 $(\pi r^2)(2r)$

V = $\frac{4}{3}\pi r^3$

Example 2.7

Find the surface area and volume of a sphere of radius 9 meters.

$SA = 4\pi r^2$

$= 4\pi(9)^2$

$= 324\pi \text{ m}^2$

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(9)^3$

$= \frac{4}{3}\pi(729)$

$= 972\pi \text{ m}^3$

Example 2.8

Find the surface area of a sphere whose volume is 288π cm^3.

SA= $4\pi r^2$

Need to find r. Get it from volume.

$$V = \frac{4}{3}\pi r^3$$

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

$$r = \sqrt[3]{\frac{3 \bullet 288\pi}{4\pi}}$$

$$r = \sqrt[3]{216}$$

r = 6 cm

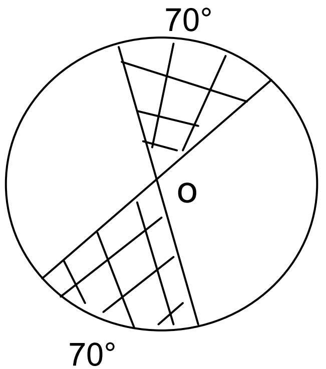
SA= $4\pi(6)^2$

= $4\pi(36)$

= 144π cm^2

Practice

a) In the circle shown, O is the center. The radius of the circle is 5 meters. Find the area of the shaded sectors.



Shaded sectors add up to 140° of the circle.

$A_{circle} = \pi r^2$

$= \pi(5)^2$

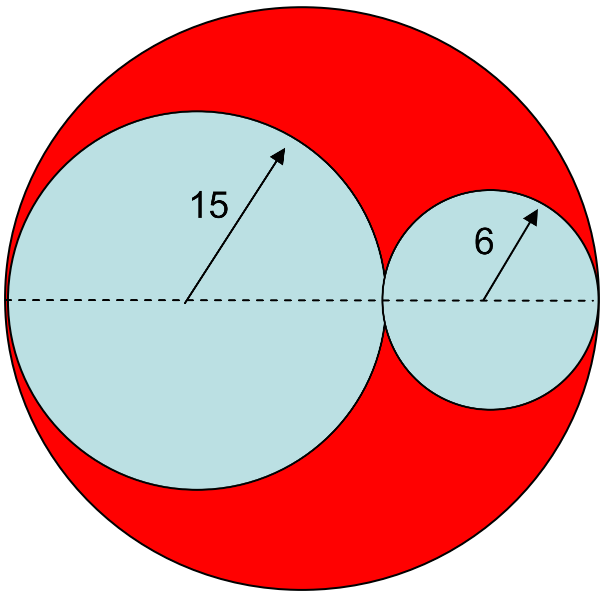
$= 25\pi\ m^2$

$A_{shaded} = 140/360\ (25\pi)$

$= 7/18\ (25\pi)$

$= 175\pi/18\ \ m^2$

b) In this figure, points C and D are the centers of two smaller circles and lie on the diameter of the big circle. The two smaller circles are tangent to the larger circle and each other. Find the area of the (red) shaded region of this figure. Dimensions are in meters.



$A_{shaded} = A_{biggest} – A_{medium} – A_{littlest}$

$A_{circle} = \pi r^2$

$A_{shaded} = \pi(21^2) – \pi(15^2) – \pi(6^2)$

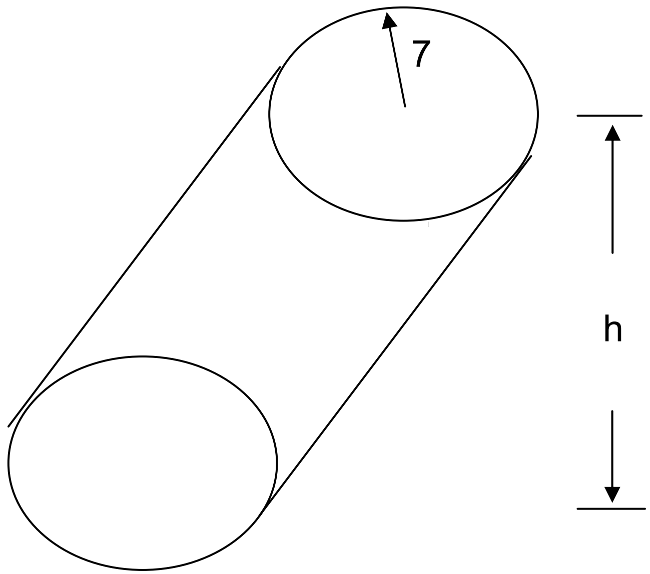
$A_{shaded} = 441\pi – 225\pi – 36\pi$

$A_{shaded} = 180\pi\ m^2$

Radius of biggest circle = ½ (15 +15 + 6 + 6)

$r_{biggest}$=21 m

c) The volume of this circular cylinder is 588π cm^3. What is the height of the circular cylinder? Dimensions are in meters.



$V_{cyl} = (A_{base})(\text{height})$

$\text{Height} = V_{cyl}/A_{base}$

$A_{base} = \pi r^2$

$= \pi(7^2)$

$= 49\pi$

$\text{Height} = 588\pi/49\pi$

$= 588/49$

$=12$ cm