

Algebra III

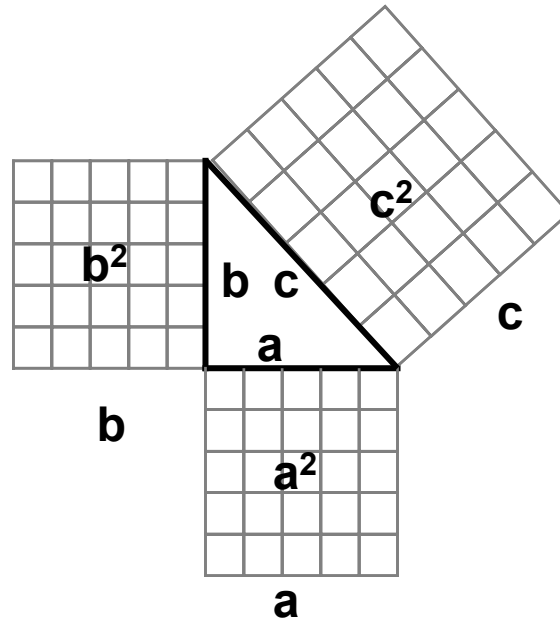
Lesson 3

**Pythagorean Theorem · Triangles Inequalities (1) ·
Similar Polygons · Similar Triangles**

Pythagorean Theorem

$$a^2 + b^2 = c^2$$

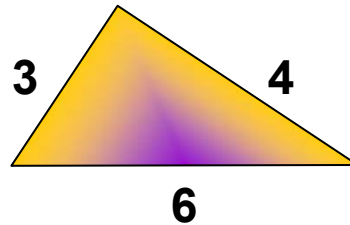
(c is always the hypotenuse – which is the longest side)



Triangle Inequalities (1)

Obtuse triangles

One angle is greater than 90°



Try Pythagorean Theorem

$$3^2 + 4^2 \ ? \ 6^2$$

$$9 + 16 \ ? \ 36$$

$$25 \ \neq \ 36$$

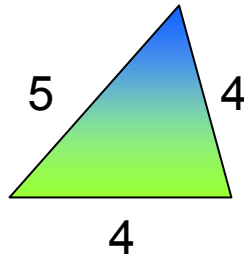
$$25 \ < \ 36$$

$$a^2 + b^2 < c^2$$

↪ Test for obtuse triangle

Acute Triangles

All angles are less than 90°



Try Pythagorean Theorem

$$4^2 + 4^2 \quad ? \quad 5^2$$

$$16 + 16 \quad ? \quad 25$$

$$32 \quad \neq \quad 25$$

$$32 \quad > \quad 25$$

$$a^2 + b^2 > c^2$$

↪ Test for obtuse triangle

Pythagoras' Statements about triangles

**Pythagorean
Theorem**

$$a^2 + b^2 = c^2$$



Test for right triangles

**Pythagorean
Inequalities**

$$a^2 + b^2 < c^2$$



Test for obtuse triangles

$$a^2 + b^2 > c^2$$



Test for acute triangles

Example 3.1

The lengths of the sides of a triangle are 7, 5, and 4, respectively.
Is the triangle a right triangle, an acute triangle, or an obtuse triangle?

Use Pythagoras' statements:

$$a^2 + b^2 \quad ? \quad c^2$$

$$4^2 + 5^2 \quad ? \quad 7^2$$

$$16 + 25 \quad ? \quad 49$$

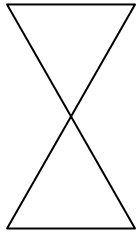
$$41 \quad < \quad 49$$

C^2 is greater so the triangle is obtuse

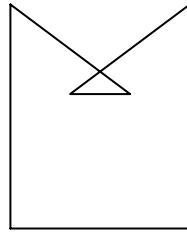
Similar Polygons

Polygon: Simple, closed, coplanar straight sides

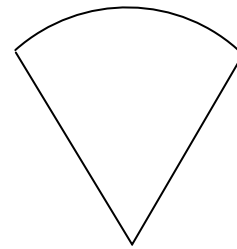
Not Polygons:



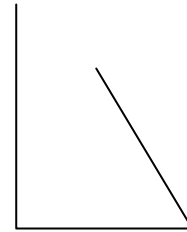
Not simple



Not simple

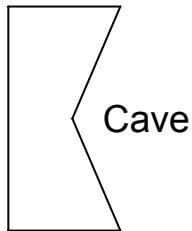


Not simple

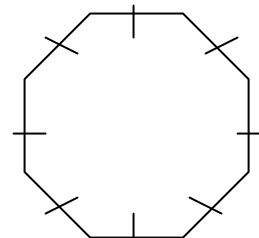
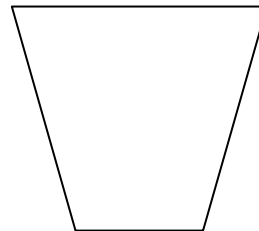
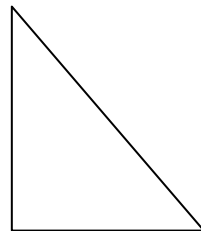


Not close

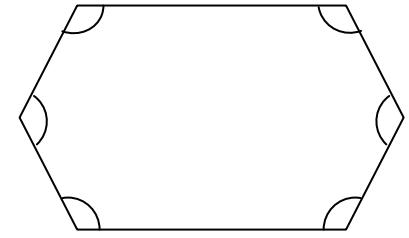
Polygons:



Concave



Equilateral



Equiangular

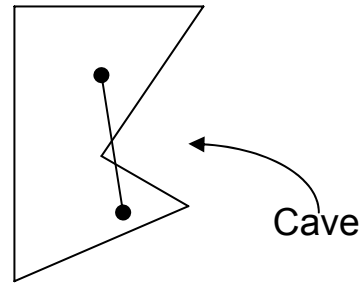
Convex

Note: Number of sides equals number of corners.

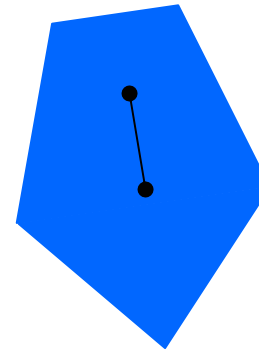
Test of Concave vs. Convex

Can a line be drawn that connects any two corners/points of the shape such that it goes outside the figure?

If so, it is concave



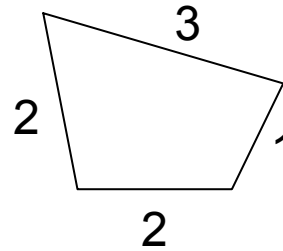
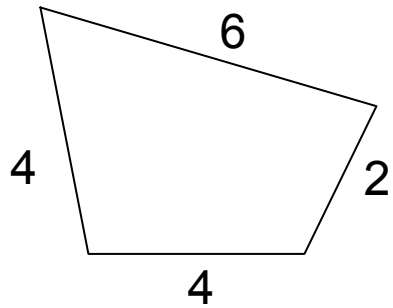
Concave
Polygon



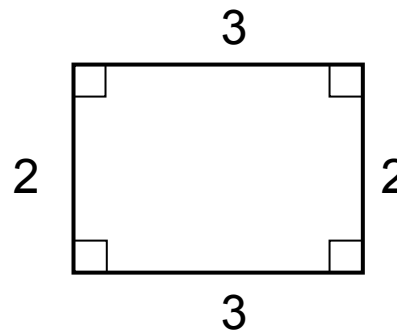
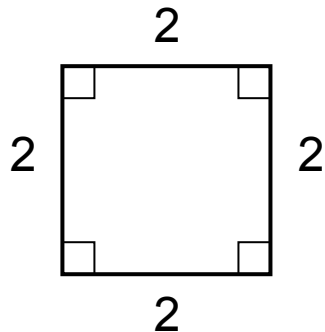
Convex
Polygon

Similar means one shape is a blown up or shrunk version of the other. Same basic shape overall.

Corresponding sides setup equal ratios



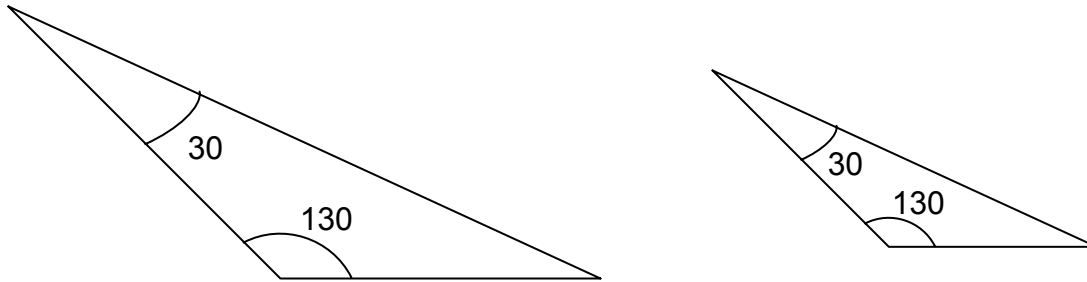
Similar



Not Similar

Similar Triangles

Corresponding angles

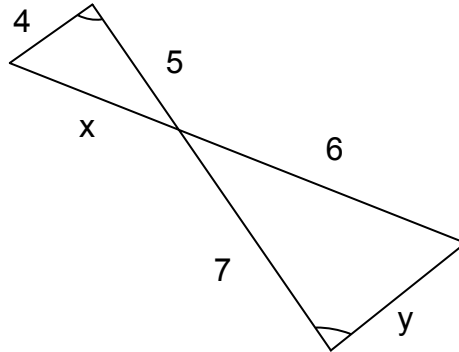


Similar triangles – all three angles are the same

(2 same means 3 same)

Example 3.2

Find x and y :



Set up ratios:

$$\frac{x}{6} = \frac{5}{7}$$

$$7x = 30$$

$$x = \frac{30}{7}$$

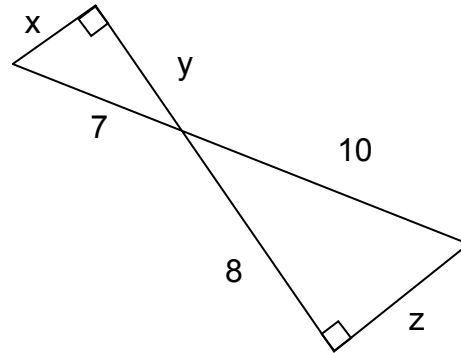
$$\frac{y}{4} = \frac{7}{5}$$

$$5y = 28$$

$$y = \frac{28}{5}$$

Example 3.3

Find x , y and z :



Find z using the Pythagorean theorem

$$10^2 = 8^2 + z^2$$

$$100 = 64 + z^2$$

$$100 - 64 = z^2$$

$$36 = z^2$$

$$6 = z$$

Set up ratios:

$$\frac{y}{8} = \frac{x}{z} = \frac{7}{10}$$

$$\frac{x}{6} = \frac{7}{10}$$

$$10x = 42$$

$$x = 4.2$$

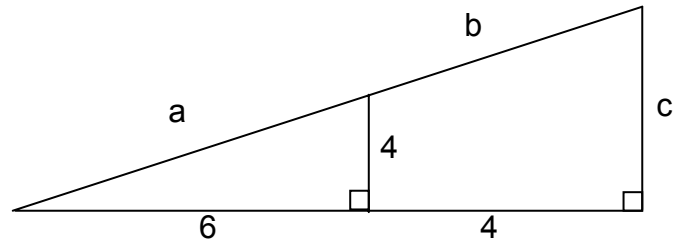
$$\frac{y}{8} = \frac{7}{10}$$

$$10y = 56$$

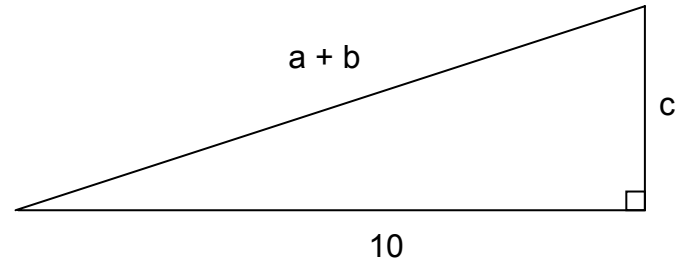
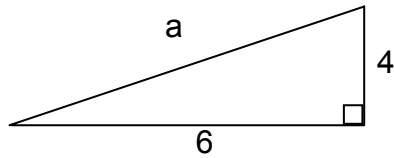
$$y = 5.6$$

Example 3.4

Find a , b and c :



Split into two triangles



Find a using
Pythagorean theorem

$$6^2 + 4^2 = a^2$$

$$36 + 16 = a^2$$

$$52 = a^2$$

$$\sqrt{52} = a$$

$$2\sqrt{13} = a$$

Set up ratios

$$10/6$$

$$c/4$$

$$(a+b)/a$$

Use a ratio to find c

$$c/4 = 10/6$$

$$6c = 40$$

$$c = 40/6 = 20/3$$

Use ratios to find b

$$\frac{a+b}{a} = \frac{10}{6}$$

$$6(a+b) = 10a$$

$$6a+6b = 10a$$

$$6b = 4a$$

$$b = 4a/6$$

$$b = 2a/3$$

$$b = \frac{2(2\sqrt{13})}{3}$$

$$b = \frac{4(\sqrt{13})}{3}$$

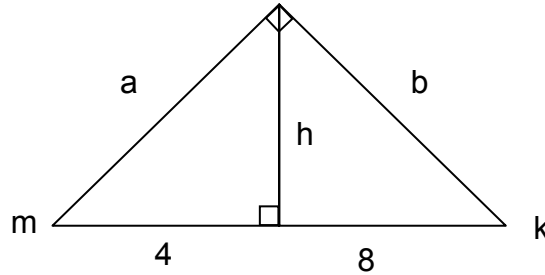
$$a = 2\sqrt{13}$$

$$b = \frac{4\sqrt{13}}{3}$$

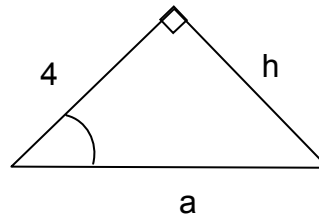
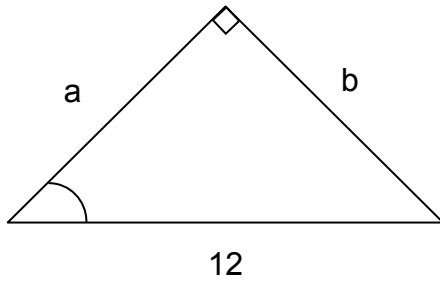
$$c = \frac{20}{3}$$

Example 3.5

Find a , b and h :



Split into two triangles



Set up ratios:

$$a/4 \quad b/h \quad 12/a$$

$$a/4 = 12/a$$

$$a^2 = 48$$

$$a = 4\sqrt{3}$$

Use Pythagorean Thm to find b and h

$$a^2 + b^2 = 12^2$$

$$4^2 + h^2 = a^2$$

$$48 + b^2 = 144$$

$$16 + h^2 = 48$$

$$b^2 = 96$$

$$h^2 = 32$$

$$b = \sqrt{96}$$

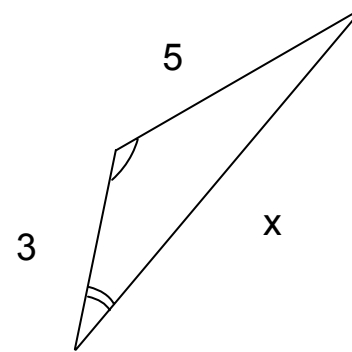
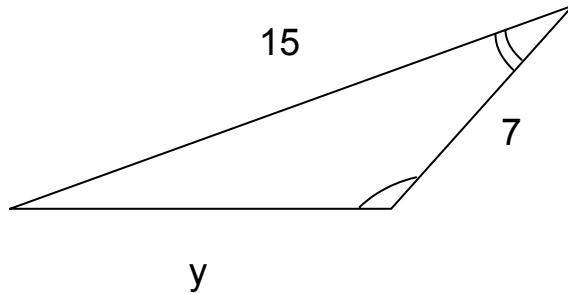
$$h = \sqrt{32}$$

$$b = 4\sqrt{6}$$

$$h = 4\sqrt{2}$$

Practice

a) Find x and y :



These are similar triangles



Setup ratio of corresponding sides

$$\frac{x}{15} = \frac{3}{7}$$

$$7x = 45$$

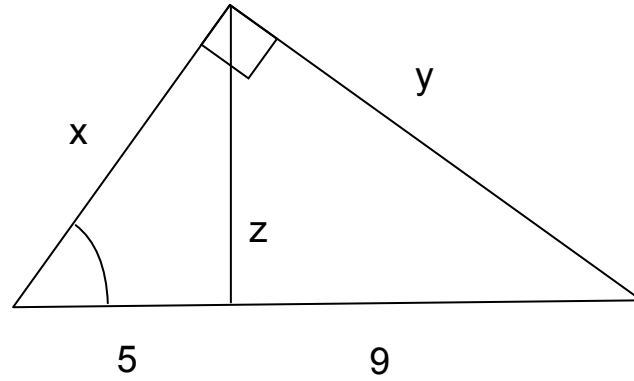
$$x = 45/7$$

$$\frac{y}{5} = \frac{7}{3}$$

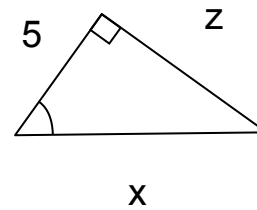
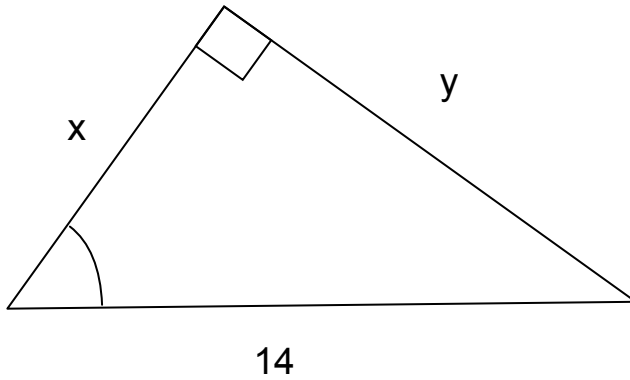
$$7y = 35$$

$$y = 35/7$$

b) Find x , y , and z



Split this into two triangles



Setup ratios of the sides

$$\frac{x}{5} = \frac{y}{z} = \frac{14}{x}$$

Find a solvable pair.

$$\frac{x}{5} = \frac{14}{x}$$

$$x^2 = 70$$

$$x = \sqrt{70}$$

Use Pythagorean Thm to find y & z.

$$x^2 + y^2 = 14^2$$

$$z^2 + 5^2 = x^2$$

$$70 + y^2 = 196$$

$$z^2 + 25 = 70$$

$$y^2 = 126$$

$$z^2 = 45$$

$$y = \sqrt{126}$$

$$z = \sqrt{45}$$

$$y = 3\sqrt{14}$$

$$z = 3\sqrt{5}$$