Algebra III

Lesson 5

Exponents & Radicals - Complex Numbers - Areas of Similar Geometric Figures - Diagonals of Rectangular Solids

Exponents & Radicals

(Nothing New)

$$\left(\sqrt{4}\right)\left(\sqrt{4}\right)=4 \qquad \left(4^{\frac{1}{2}}\right)\left(4^{\frac{1}{2}}\right)=4^{\frac{1}{2}+\frac{1}{2}}=4^{1}=4$$

Example 5.1

$$Simplify:\ \left(\sqrt{x^{3}y}\right)\left(\sqrt[4]{xy^{3}}\right)$$

$$=\left(x^{3}y\right)^{\frac{1}{2}}\left(xy^{3}\right)^{\frac{1}{4}}$$

$$=x^{\frac{3}{2}}y^{\frac{1}{2}}x^{\frac{1}{4}}y^{\frac{3}{4}}$$

$$For\ \mathrm{x}:\ \frac{3}{2}+\frac{1}{4}=\frac{6}{4}+\frac{1}{4}=\frac{7}{4}$$

$$For\ \mathrm{y}:\ \frac{1}{2}+\frac{3}{4}=\frac{2}{4}+\frac{3}{4}=\frac{5}{4}$$

Putting this together gives: $x^{\frac{7}{4}}y^{\frac{5}{4}}$

Example 5.2

$$\textit{Simplify}: \frac{\mathrm{a}^{\frac{\mathrm{x}}{2}}\left(y^{2-x}\right)^{\frac{1}{2}}}{a^{3x}y^{-2x}}$$

$$= \frac{\mathrm{a}^{\frac{\mathrm{x}}{2}}\left(y^{\frac{1}{2}(2-x)}\right)}{a^{3x}y^{-2x}}$$

$$= a^{\frac{x}{2}}a^{-3x}y^{1-\frac{x}{2}}y^{2x}$$

$$= a^{\frac{x}{2}-3x}y^{1-\frac{x}{2}+2x}$$

$$= a^{-\frac{5x}{2}}y^{1+\frac{3x}{2}}$$

$$= a^{-\frac{5x}{2}}y^{\frac{2+3x}{2}}$$

Example 5.3

$$Simplify: \frac{x^{-2}+y^{-2}}{(xy)^{-1}}$$

$$=\left(x^{-2}+y^{-2}\right)(xy)$$

$$=x^{-1}y+xy^{-1}$$

$$=\frac{y}{x}+\frac{x}{y}$$

$$=\frac{y^2}{xy}+\frac{x^2}{xy}$$

$$=\frac{y^2+x^2}{xy}$$

Example 5.4

$$Simplify:\ 3\sqrt{\frac{3}{2}}-4\sqrt{\frac{2}{3}}+2\sqrt{24}$$

$$=3\sqrt{\frac{3}{2}\cdot\frac{2}{2}}-4\sqrt{\frac{2}{3}\cdot\frac{3}{3}}+2\sqrt{4\cdot 6}$$

$$=\frac{3}{2}\sqrt{6}-\frac{4}{3}\sqrt{6}+4\sqrt{6}$$

Adding coefficients:

$$\frac{3}{2}-\frac{4}{3}+4=\frac{9}{6}-\frac{8}{6}+\frac{24}{6}=\frac{25}{6}$$

$$\text{Resulting in}:\ \frac{25\sqrt{6}}{6}$$

Example 5.5

$$\textit{Simplify}: \left(\sqrt{2}+\sqrt{x}\right)\left(1-\sqrt{x}\right)$$

$$=\left(\sqrt{2}\right)(1)-\left(\sqrt{2}\right)\left(\sqrt{x}\right)+\left(\sqrt{x}\right)(1)-\left(\sqrt{x}\right)\left(\sqrt{x}\right)$$

$$=\sqrt{2}-\sqrt{2x}+\sqrt{x}-x$$

$$=\sqrt{2}+\sqrt{x}-\sqrt{2}\sqrt{x}-x$$

$$=\sqrt{2}+\left(1-\sqrt{2}\right)\sqrt{x}-x$$

Complex Numbers
(Nothing New)

Example 5.6

$$Simplify: \; 3i^3 + 2i^5 - 3i + 2i^2$$

$$= 3i^2 i + 2i^4 i - 3i + 2i^2$$

$$= 3(-1)i + 2(1)i - 3i + 2(-1)$$

$$= -3i + 2i - 3i - 2$$

$$= -2 - 4i$$

Example 5.7

$$\textit{Simplify}: \sqrt{-3}\sqrt{4}+3\sqrt{-2}\sqrt{-9}+\sqrt{-16}+\sqrt{16}$$

$$=\sqrt{3}i\sqrt{4}+3\sqrt{2}i\sqrt{9}i+\sqrt{16}i+\sqrt{16}$$

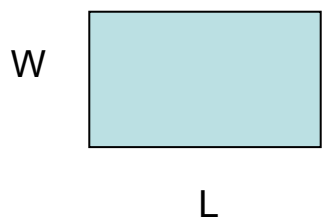
$$=2\sqrt{3}i-9\sqrt{2}+4i+4$$

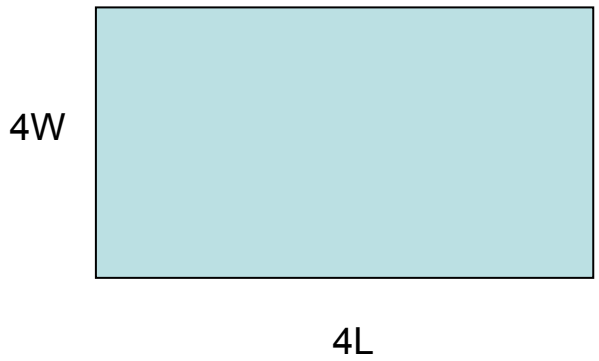
$$=\left(4-9\sqrt{2}\right)+\left(4+2\sqrt{3}\right)i$$

Areas of Similar Geometric Figures

Example 5.8

The scale factor between two rectangles is 4. What is the ratio of the areas?





$A_1 = LW$

$A_2 = (4L)(4W)$

$= 16LW$

$A_2 = 16A_1$

So, the ratio of the areas is 16/1 or 1/16.

Example 5.9

The radius of one circle is R, and the radius of a second circle is 3R/5. What is the ratio of the area of the first circle to the area of the second circle?

$A = \pi r^2$

The ratio of the 1st to the 2nd means $\rightarrow A_1/A_2$.

$A_1 = \pi R^2$

$A_2 = \pi(3R/5)^2$

$= (9\pi R^2)/25$

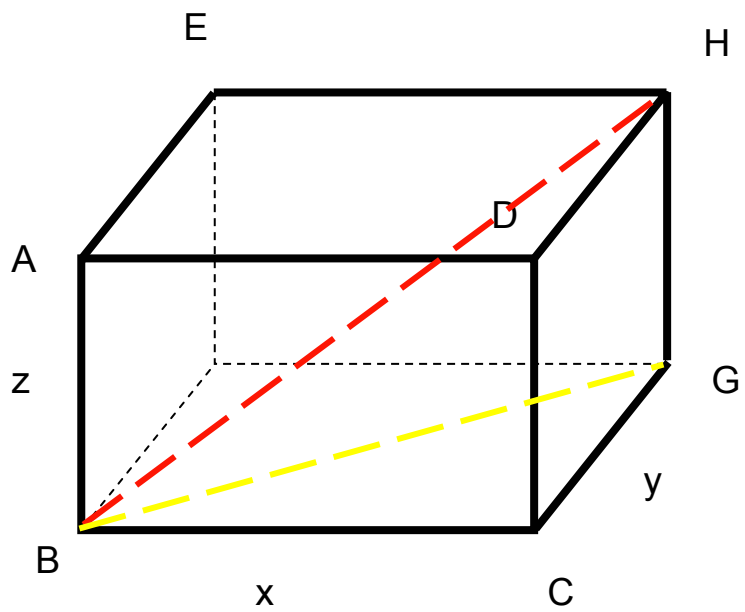
$$\frac{A_1}{A_2} = \frac{\pi R^2}{\frac{9\pi R^2}{25}}$$

$$\text{So, } \frac{A_1}{A_2} = \frac{25}{9}$$

Diagonals of Rectangular Solids

Example 5.10

Find the length of the diagonal that connects corners B & H in the rectangular solid shown.



Notice that BH (in red) and HG are two sides of a right triangle, missing only BG (in yellow).

So, by Pythagorean Thm:

$BH^2 = HB^2 + BG^2$

Note that HB = z, but BG must be found by Pythagorean Thm and triangle BCG.

$BG^2 = BC^2 + CG^2$

$BG^2 = x^2 + y^2$

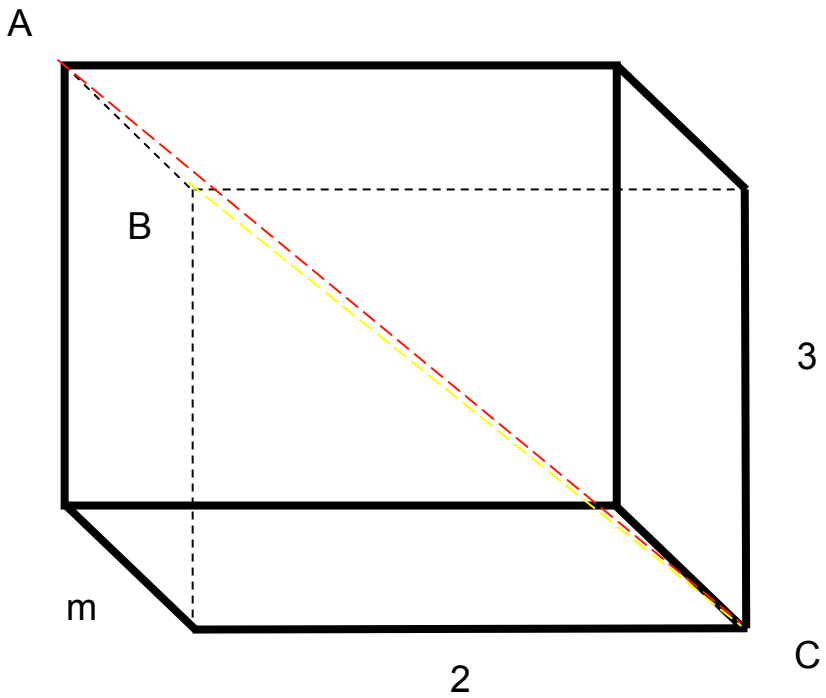
Sub this into the $BH^2 = HB^2 + BG^2$

$BH^2 = z^2 + x^2 + y^2$

$$\text{So, } BH = \sqrt{x^2 + y^2 + z^2}$$

Example 5.11

Find the length of diagonal AC in the rectangular solid shown. Do a two-step development, do not use the formula from 5.10.



$BC^2 = 3^2 + 2^2$

$BC^2 = 13$

$$BC = \sqrt{13}$$

$AC^2 = BC^2 + m^2$

$AC^2 = 13 + m^2$

$$AC = \sqrt{13 + m^2}$$

AC is in red. BC is in yellow.
Need to find BC to find AC.

Practice

$a)$ *Simplify* : $\dfrac{\mathrm{a}^{\frac{\mathrm{x}}{2}}\left(y^{2-x}\right)^{\frac{1}{2}}}{a^{4x}y^{-2x}}$

$$= \frac{\mathrm{a}^{\frac{\mathrm{x}}{2}}\left(y^{\frac{1}{2}(2-x)} \right)}{a^{4x}y^{-2x}}$$

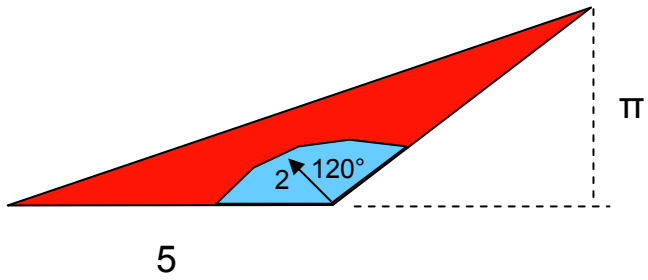
$$= a^{\frac{x}{2}}a^{-4x}y^{1-\frac{x}{2}}y^{2x}$$

$$= a^{\frac{x}{2}-4x}\ y^{1-\frac{x}{2}+2x}$$

$$= a^{-\frac{7x}{2}}\ y^{1+\frac{3x}{2}}$$

$$= a^{-\frac{7x}{2}}\ y^{\frac{2+3x}{2}}$$

b) Find the area of the **shaded** region. Dimensions are in meters.



$A_{shaded} = A_{triangle} - A_{part\ of\ circle}$

$A_{tri} = \frac{1}{2}bh$

$= \frac{1}{2}(5)(\pi)$

$= \frac{5\pi}{2}$

$A_{part\ of\ circle} = \left(\frac{arc}{360}\right)\pi r^2$

$= \left(\frac{120}{360}\right)\pi(2)^2$

$= \frac{4\pi}{3}$

$A_{shaded} = \frac{5\pi}{2} - \frac{4\pi}{3}$

$= \frac{15\pi}{6} - \frac{8\pi}{6}$

$A_{shaded} = \frac{7\pi}{6}\ \text{m}^2$