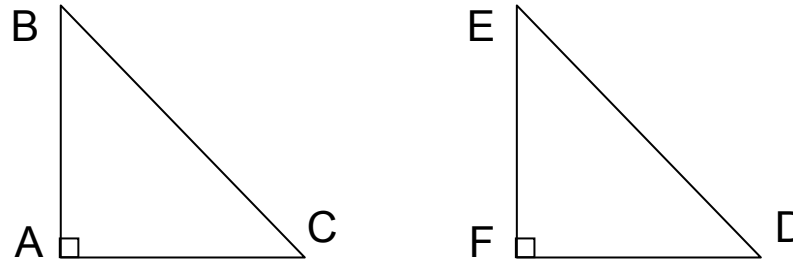


Algebra III

Lesson 8

**Statements of Similarity
– Proportional Segments –
Angle Bisectors and Side Ratios**

Statement of Similarity



$$\triangle ABC \sim \triangle FED$$

Letters must follow in exact order

$$\triangle CBA \sim \triangle DEF$$

$$\triangle ABC \sim \triangle DFE \quad (\text{Not similar})$$

Setting up ratios from statement of similarity

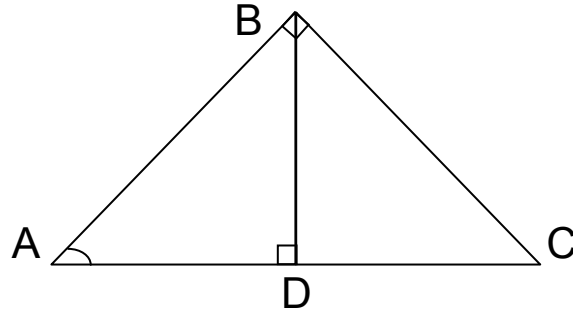
$$\triangle ABC \sim \triangle FED$$

$$\frac{AB}{FE} \quad \frac{BC}{ED} \quad \frac{AC}{FD}$$

Example 8.1

In $\triangle ABC$, segment BD is the altitude to the hypotenuse AC

Show the following: $\frac{AD}{AB} = \frac{AB}{AC}$



Make a statement of similarity

$$\triangle ADB \sim \triangle ABC$$

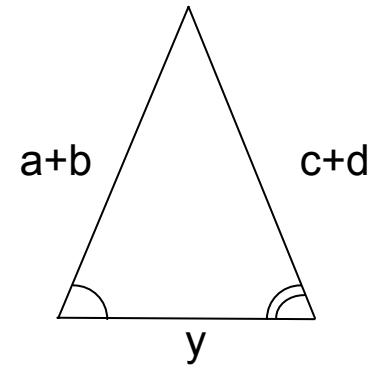
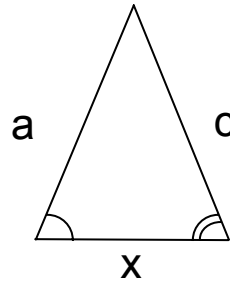
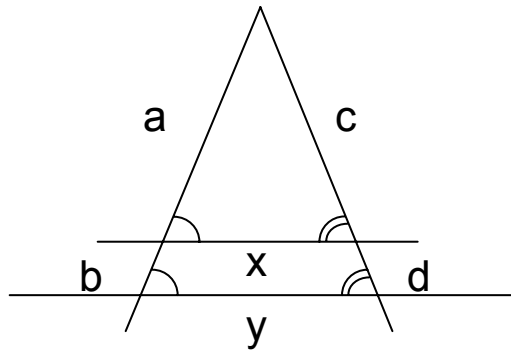
Get the ratios

$$\frac{AD}{AB} \quad \frac{DB}{BC} \quad \frac{AB}{AC}$$

$$\frac{AD}{AB} = \frac{AB}{AC}$$

Proportional segments (reproving something already doing)

Two triangles that are similar



Get ratios of: $\frac{a+b}{a}$ $\frac{c+d}{c}$ $\frac{y}{x}$

$$\frac{a+b}{a} = \frac{c+d}{c}$$

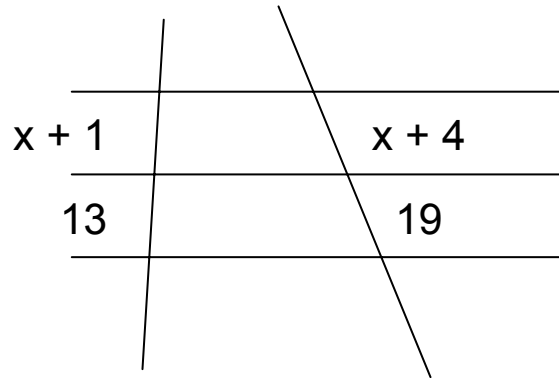
$$\frac{a}{a} + \frac{b}{a} = \frac{c}{c} + \frac{d}{c}$$

$$1 + \frac{b}{a} = 1 + \frac{d}{c}$$

$$\frac{b}{a} = \frac{d}{c}$$

Example 8.2

Find x



$$\frac{x + 1}{13} = \frac{x + 4}{19}$$

$$19(x + 1) = 13(x + 4)$$

$$19x + 19 = 13x + 52$$

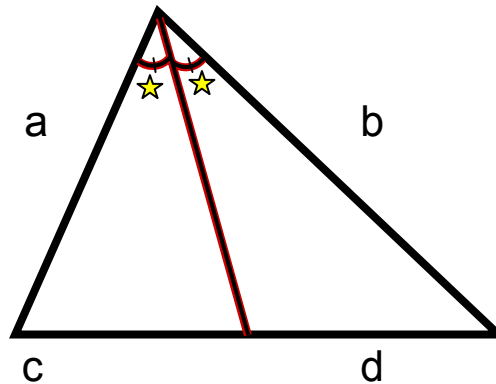
$$6x = 33$$

$$x = \frac{33}{6} = \frac{11}{2}$$

Angle Bisectors and side ratios

Start with a triangle

Bisect one angle



The two stars emphasize that the angle has been bisected.

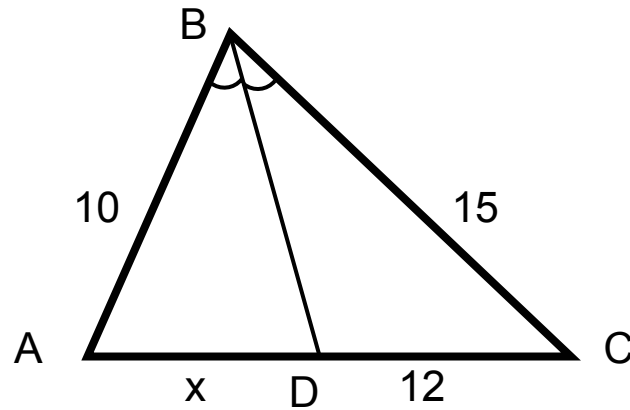
So you can make a ratio of sides in two ways

$$\frac{a}{c} = \frac{b}{d} \quad \text{or} \quad \frac{a}{b} = \frac{c}{d}$$

Example 8.3

In $\triangle ABC$, segment BD is the angle bisector of angle B

Find x



$$\frac{x}{10} = \frac{12}{15}$$

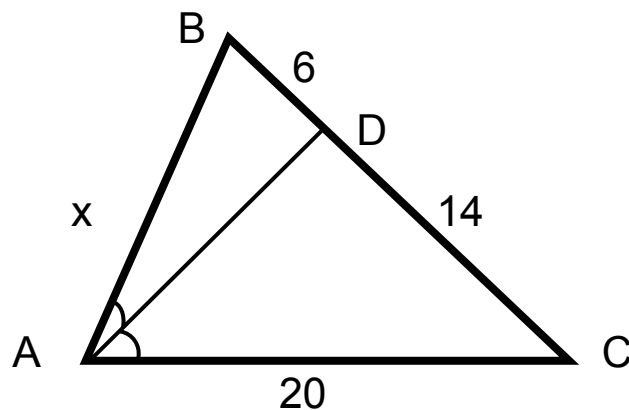
$$15x = 120$$

$$x = 8$$

Example 8.4

- a) In $\triangle ABC$, segment AD is the angle bisector of angle A

Find x



$$\frac{x}{20} = \frac{6}{14}$$

$$14x = 120$$

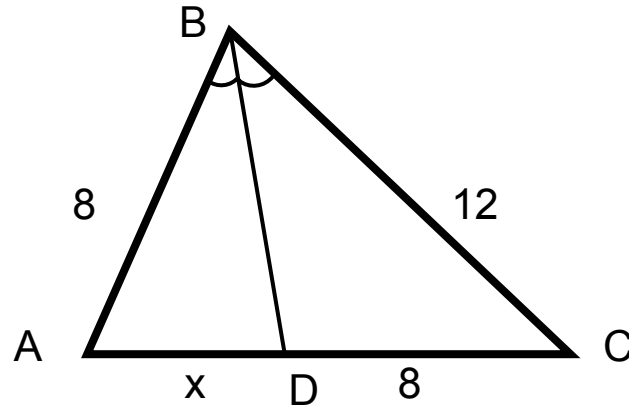
$$x = \frac{120}{14}$$

$$= \frac{60}{7}$$

Practice

a) In $\triangle ABC$, \overline{BD} is the angle bisector of angle B

Find x



$$\frac{x}{8} = \frac{8}{12}$$

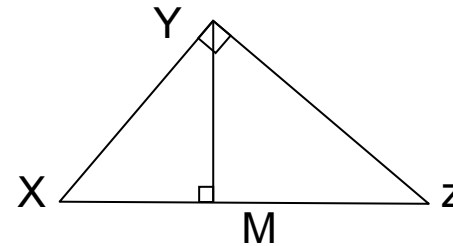
$$12x = 64$$

$$x = \frac{64}{12}$$

$$= \frac{16}{3}$$

b) Given $\triangle XYZ$ is a right triangle \overline{YM} is the altitude to the hypotenuse of XZ

Show the following: $\frac{XY}{XM} = \frac{YZ}{MY}$



Make two similar triangles $\triangle XMY \sim \triangle YMZ$

Set up ratios $\frac{XM}{XY} = \frac{MY}{YZ} = \frac{XY}{XZ}$

Flip them over $\frac{XY}{XM} = \frac{YZ}{MY} = \frac{XZ}{XY}$

Now pick two needed $\frac{XY}{XM} = \frac{YZ}{MY}$

c) Is the following argument valid or invalid?

All authors wear gold watches
John wears a gold watch
∴ John is an author

Invalid: Follows major premise backwards