Algebra III

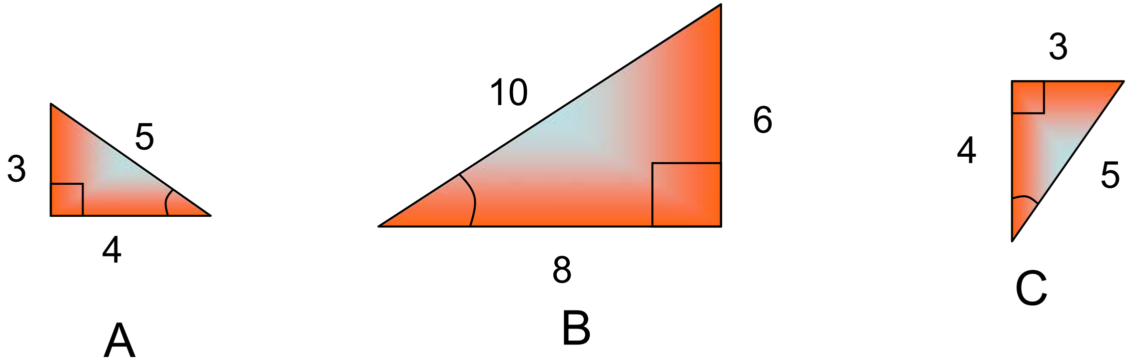
Lesson 9

Congruent Figures – Proof Outlines

Congruent Figures

All aspects of two geometric figures are equivalent.

If one figure can be placed such that it exactly matches another figure, then the two figures are congruent.



$A \sim B \sim C$

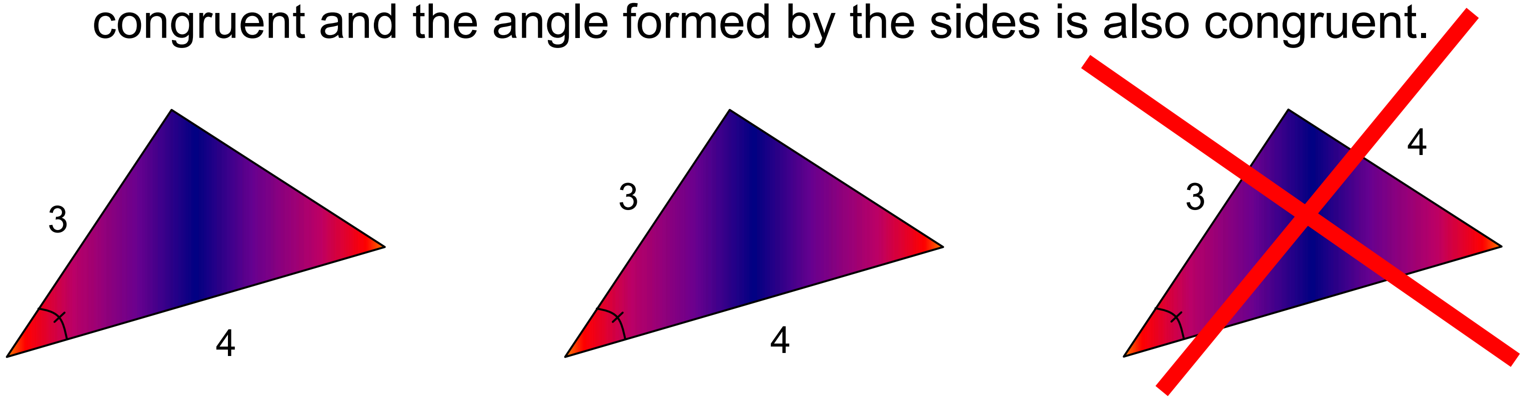
$A \cong B$

4 Ways to Prove Triangles Congruent

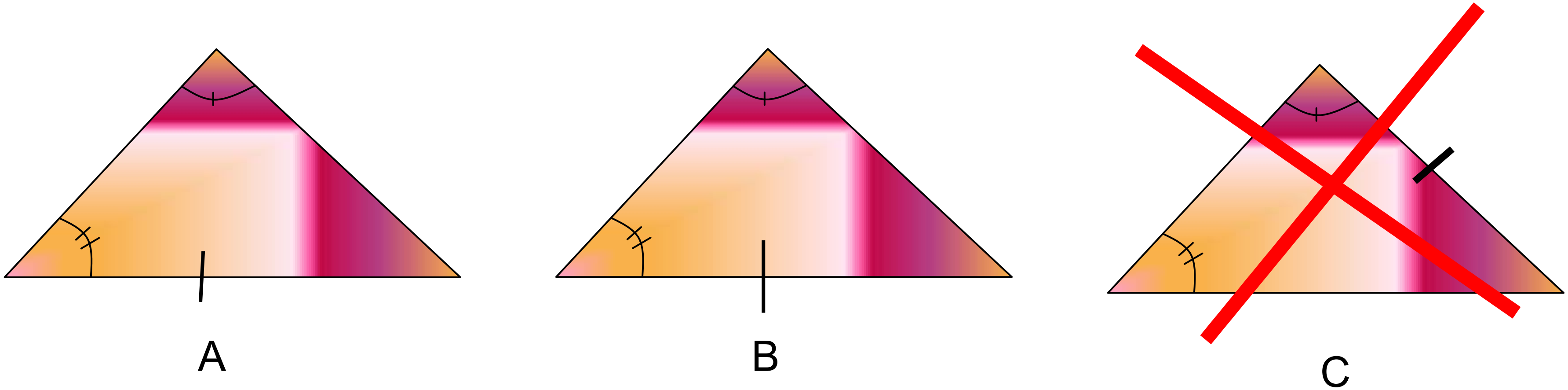
We will not now prove that they are true, we will just accept them as true, or as **postulates.**

1) SSS → side-side-side → all three sides are exactly the same.

2) SAS → side-angle-side → Two sides of the triangle are congruent and the angle formed by the sides is also congruent.



3) ASA → angle-side-angle → This requires two angles and one side to be the same. Warning: the sides must be in corresponding positions. (A.K.A.: AAAS)



A ≅ B, but C is not congruent to either one since the equivalent side is in the wrong spot.

4) HL → hypotenuse-leg → in a right triangle the hypotenuse and one leg must be the same.

Note: Figures are not equal (□ = □), they are congruent (□ $\cong$ □).

Also, measurements are not congruent ($\angle A \cong 50°$), they are equal ($\angle A = 50°$)

Proof Outlines

These are not true, full blown, agonizing geometric proofs.

They ore initial, basic steps needed to do a true geometric proof. And only mildly irritating at best.

When $\angle ABC \cong \angle DEF$ is written, then that is the way they must match up. $\angle B$ is the same as $\angle E$, B & E, C & F. Also the sides must match up, AB & DE, etc.

Some Symbols:

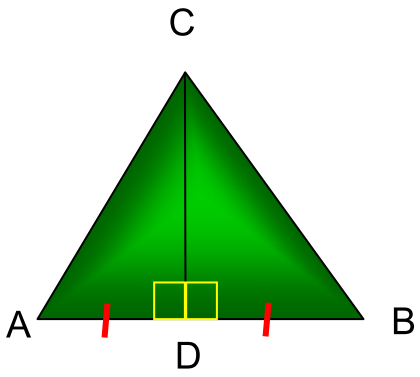
$\perp$ → Perpendicular

$\|$ → Parallel

Example 9.1

Given: $\overline{AD} \cong \overline{BD}$

$\overline{DC} \perp \overline{AB}$



Outline a proof that shows:

$\overline{AC} \cong \overline{BC}$

Since, $\overline{AC}$ & $\overline{BC}$ are corresponding sides in Δ's ACD & BCD, if the triangles can be proven congruent then the corresponding sides are congruent.

How to prove the triangles congruent:

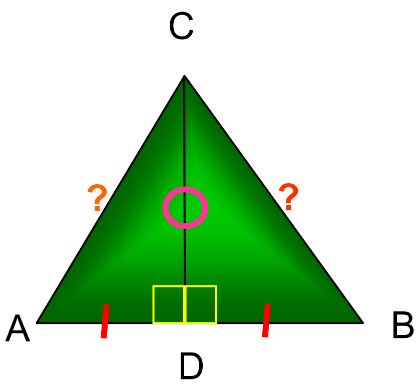
-- Make notes on the figure about what parts are given as congruent

$\overline{AD} \cong \overline{BD}$ (in red)

$\overline{DC} \perp \overline{AB}$ (in yellow)

-- Add any logical/quick conclusions about the figures (none here)

After the making of notes, it is time to try to find a way to prove the two triangles congruent.



Remember what is to be **proven**

-- Since this is a right triangle I'll try the **HL** postulate. This won't work because while we know a leg we don't know about the hypotenuses (in fact this is what is to be proven).

-- Next, I'll try SSS → won't work since we are trying to prove one of the sides

-- Now, try SAS → we have one side (the bottoms) and an angle (the right angles), if the next side in order can be found congruent then we can use this. Since in both triangles $\overline{BD}$ is the next side, we can say $\overline{BD} \cong \overline{BD}$, which allow use to prove the triangles congruent. (Noted in pink.)

-- So now we can say: $\Delta ACD \cong \Delta BCD$ SAS

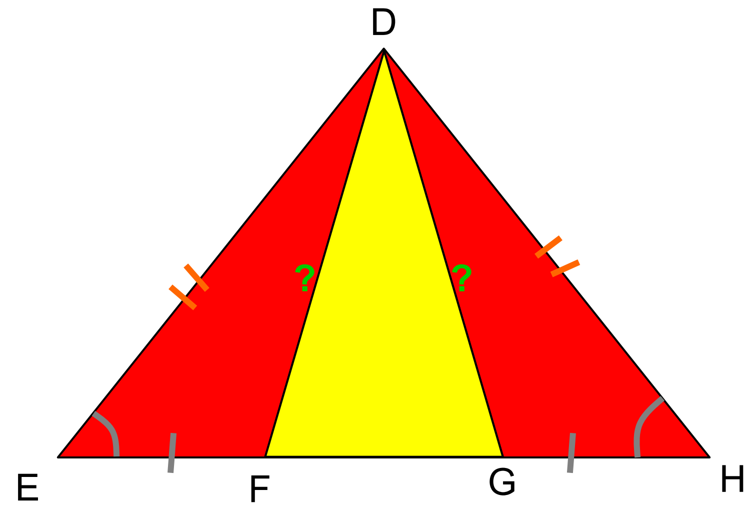
$\therefore \overline{AC} \approx \overline{BC}$ CPCTC

CPCTC → **C**ongruent **P**arts of **C**ongruent **T**riangles are **C**ongruent

Example 9.2

Given: $\angle E \cong \angle H$

$\overline{EF} \cong \overline{HG}$



Outline a proof that shows:

$$\overline{DF} \cong \overline{DG}$$

-- Start by marking up the figure with the **givens** and what is to be **proved**.

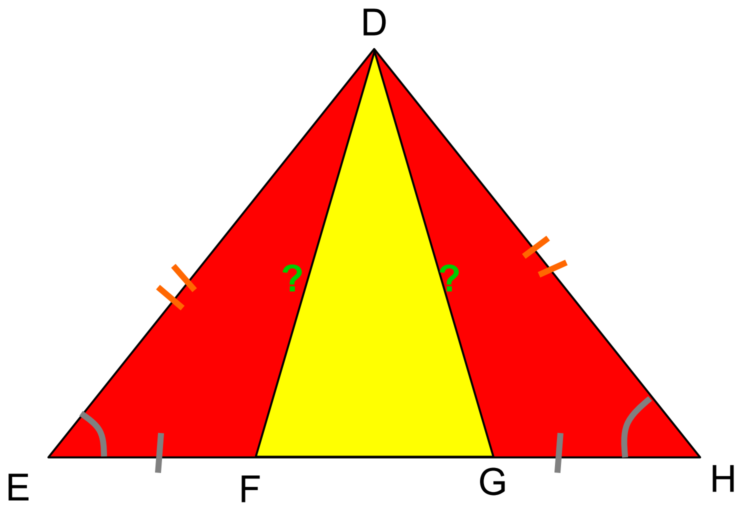
-- Look at the figure to determine the triangles to be worked with. I see ΔEFD and ΔHGD. Shown by the two little **red** triangles on the sides.

-- Look at figure to see if any additional information can be determined from the givens. The big red ΔEDH has the two angles the same, that makes it an isosceles triangle. Which makes $\overline{ED} \cong \overline{HD}$ (**in orange**).

-- Figure out the way to prove the two triangles congruent.

** SSS – Nope, don't have all three sides.

** SAS – Yes, this will work.



So, $\Delta EFD \cong \Delta HGD$ SAS

$\therefore \overline{DF} \cong \overline{DG}$ CPCTC

Example 9.3

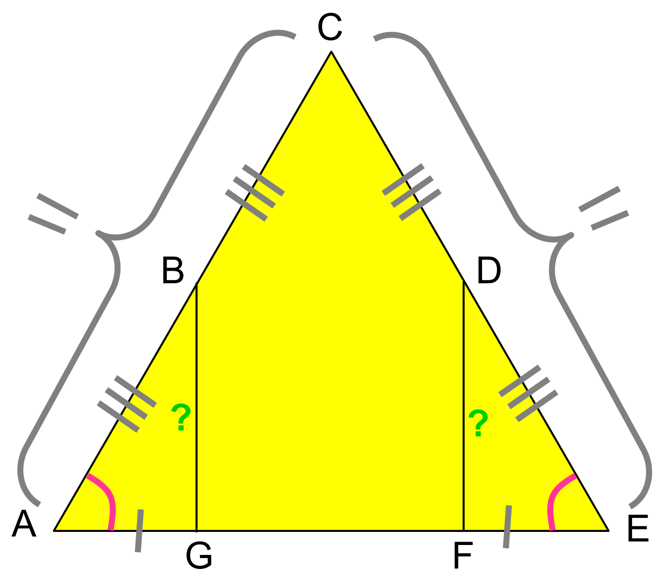
In this problem we will remember that if two segments have equal lengths, then the halves of the segments have equal lengths.

Given: $\overline{AG} \cong \overline{EF}$

$\overline{CA} \cong \overline{HG}$

B is the midpoint of $\overline{CA}$

D is the midpoint of $\overline{CE}$



Outline a proof that shows:

$\overline{BG} \cong \overline{DF}$

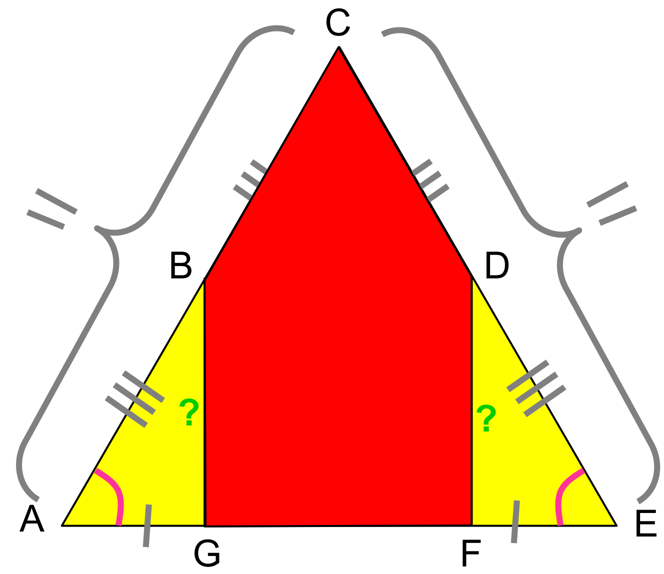
-- Fill in the **givens** and the **proves** on the figure.

-- Fill in any obvious info. Since the two sides of the big triangle are equal that makes it an isosceles triangle, which makes $\angle A \cong \angle E$. **(in pink)**

-- Find the triangles that need to be proven congruent.

ΔABG ≅ ΔEDF

Now the two small triangles in yellow.



-- Find the way to prove them congruent.

SSS - NO

SAS -Yes

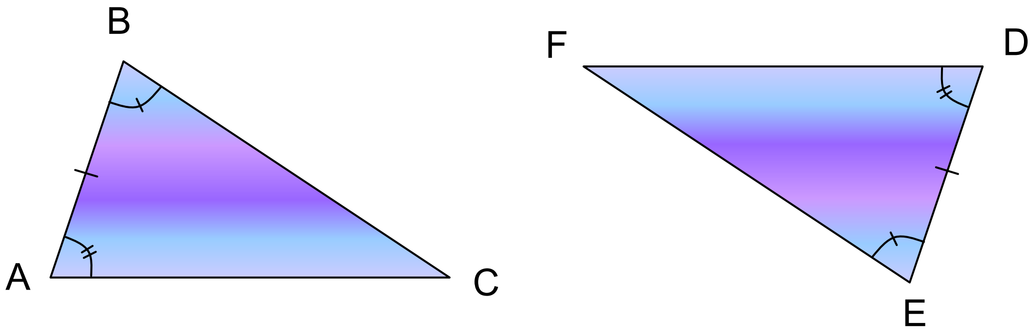
So, $\Delta ABG \cong \Delta EDF$ SAS

$\therefore \overline{BG} \cong \overline{DF}$ CPCTC

Practice

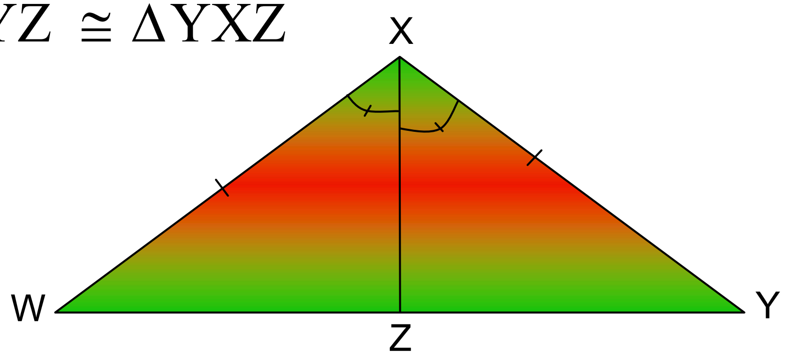
a) Triangles can be shown to be congruent by the SSS congruency postulate, SAS congruency postulate, AAAS congruency postulate (ASA), or the HL congruency postulate. The triangles in each pair shown below are congruent. State the congruency postulate that can be used to prove them congruent.

a) $\Delta \mathrm{A}BC \cong \Delta \mathrm{D}EF$



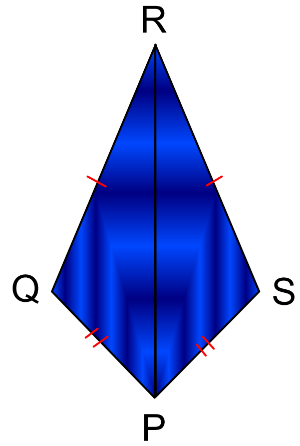
ASA or AAAS

b) $\Delta \mathrm{WYZ} \cong \Delta \mathrm{YXZ}$



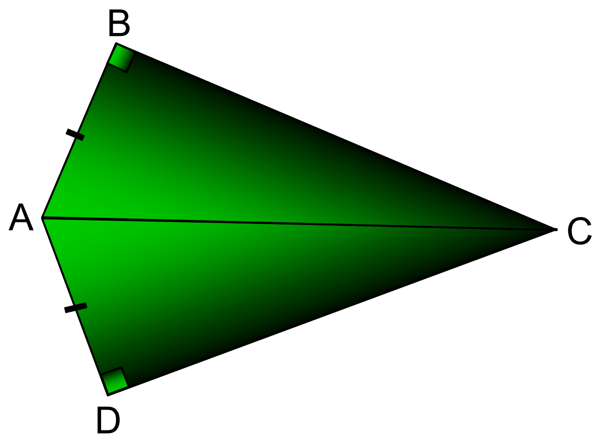
SAS

c) $\Delta PQR \cong \Delta PSR$



SSS

d) $\Delta ABC \cong \Delta ADC$



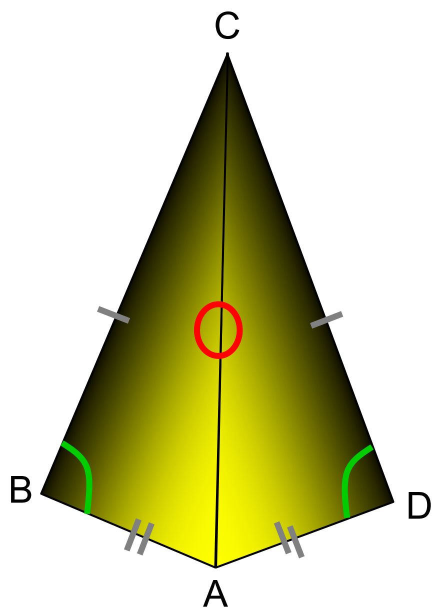
HL

b) Given: $\overline{BC} \cong \overline{DC}$

$\overline{AB} \cong \overline{AD}$

Outline a proof that shows:

$\angle ABC \cong \angle ADC$



-- Mark givens and **proves** on the figure.

-- Fill in anything else obvious.

-- Find Δ's -- ΔABC & ΔADC

-- Find a postulate

a) SSS – works since the two triangles share $\overline{CA}$. (in red)

So, $\Delta ABC \cong \Delta ADC$	SSS
$\therefore \angle ABC \cong \angle ADC$	CPCTC