

Algebra III

Lesson 10

Equation of a Line – Rational Denominators
– Completing the Square

Nothing New

Equation of a line

Horizontal line – Slope = 0 \rightarrow form $y = \square$

Vertical line – No slope or undefined \rightarrow form $x = \square$

– No y intercept

Slope – intercept form of line

$$y = mx + b$$

$$m = \text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_1 - y_2}{x_1 - x_2}$$

$b = y$ intercept

Example 10.1

Find the slope intercept form of the equation of the line that passes through the points $(-3,2)$ & $(4, -3)$.

Remember: $y = mx+b$

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{2 - (-3)}{-3 - 4} = \frac{5}{-7} = -\frac{5}{7}$$

$$y = -\frac{5}{7}x + b$$

Use one of the points to find b : $(-3, 2)$

$$2 = -\frac{5}{7}(-3) + b$$

$$2 = \frac{15}{7} + b$$

$$b = 2 - \frac{15}{7}$$

$$= -\frac{1}{7}$$

$$\therefore y = -\frac{5}{7}x - \frac{1}{7}$$

Example 10.2

Find the equation of the line that passes through the point $(-2, 5)$ and is perpendicular to the line $3y+4x = 2$

Perpendicular to a line means the negative inverse of the slope of the given line.

$$\text{So, if } 3y+4x = 2$$

$$3y = -4x+2$$

$$y = -4/3x+2/3$$

$$m = -4/3$$

∴ For the new line $m = 3/4$

$$\text{So, } y = \frac{3}{4}x + b$$

Use the point (-2,5) to find b

$$5 = \frac{3}{4}(-2) + b$$

$$5 = -\frac{6}{4} + b$$

$$5 + \frac{3}{2} = b$$

$$\frac{10}{2} + \frac{3}{2} = b$$

$$\frac{13}{2} = b$$

∴ The equation of the new line is $y = \frac{3}{4}x + \frac{13}{2}$

Example 10.3

Find the equation of lines (a) and (b)

a) Horizontal line

$$y = \square$$

$$y = -2$$

b) $y = mx + b$

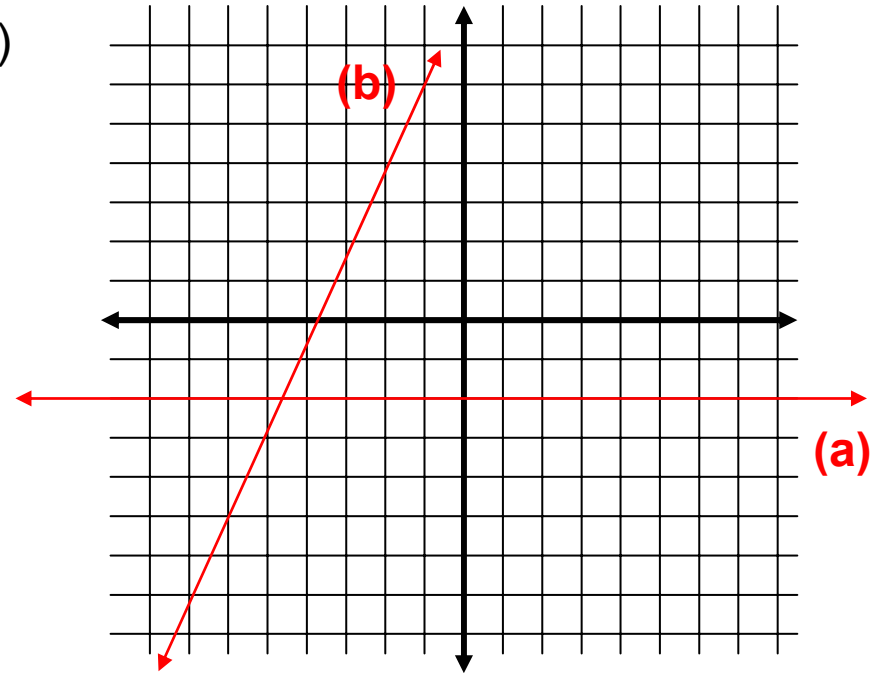
Find two points: $(-1, 6)$ $(-6, -5)$

$$\text{Find slope: } m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{6 - (-5)}{-1 - (-6)} = \frac{11}{5}$$

$$y = 11/5 x + b$$

Use a point to find b. $(-1, 6)$

$$6 = 11/5(-1) + b$$



$$6 = -11/5 + b$$

$$6 + 11/5 = b$$

$$41/5 = b$$

$$y = 11/5 x + 41/5$$

Rational Denominators

Remember rational means a number that ends or repeats

2

4.7

5.234234...

6.6666...

Note any root that doesn't have a nice answer is irrational.
Imaginary numbers cannot be rational.

The denominators of fractions will be rational.

Example 10.4

Simplify: $\frac{4 + \sqrt{3}}{2 - 3\sqrt{3}}$

Clear this by using the conjugate of the denominator

$$2 + 3\sqrt{3}$$

$$\frac{4 + \sqrt{3}}{2 - 3\sqrt{3}} \cdot \frac{2 + 3\sqrt{3}}{2 + 3\sqrt{3}}$$

$$= \frac{8 + 12\sqrt{3} + 2\sqrt{3} + 9}{4 + 6\sqrt{3} - 6\sqrt{3} - 27}$$

$$= \frac{17 + 14\sqrt{3}}{-23} \text{ or } \frac{-17 - 14\sqrt{3}}{23}$$

Example 10.5

Simplify: $\frac{2 - i^3 + 2i^5}{-2i + 4}$

1st take care of the top $\rightarrow 2 - i^2i + 2i^4i$

$$= 2 + i + 2i$$

$$= 2 + 3i$$

2nd rearrange bottom $\rightarrow -2i + 4$

$$= 4 - 2i$$

Now put back together

$$\frac{2 + 3i}{4 - 2i}$$

Clear i from the bottom by the conjugate

$$\begin{aligned} & 4 + 2i \\ & \frac{2 + 3i}{4 - 2i} \cdot \frac{4 + 2i}{4 + 2i} \\ & = \frac{8 + 4i + 12i + 6i^2}{16 + 8i - 8i - 4i^2} \\ & = \frac{2 + 16i}{20} \\ & = \frac{1}{10} + \frac{8}{10}i \\ & = \frac{1}{10} + \frac{4}{5}i \end{aligned}$$

Completing the Square

Enough Said.

Example 10.6

Solve $-x + 3x^2 = -5$ by completing the square.

Rearrange

$$3x^2 - x = -5$$

Multiply by $\frac{1}{3}$ to make x^2

$$x^2 - \frac{1}{3}x = -\frac{5}{3}$$

Half the x term

$$\left(-\frac{1}{3}\right)\left(\frac{1}{2}\right) = -\frac{1}{6}$$

Square this.

$$\left(-\frac{1}{6}\right)^2 = \frac{1}{36}$$

Add this to both sides

$$x^2 - \frac{1}{3}x + \frac{1}{36} = -\frac{5}{3} + \frac{1}{36}$$

Simplify both sides.

$$-\frac{5}{3} + \frac{1}{36} = -\frac{60}{36} + \frac{1}{36} = -\frac{59}{36}$$

$$\left(x - \frac{1}{6}\right)^2 = -\frac{59}{36}$$

Square root both sides

$$x - \frac{1}{6} = \pm \sqrt{\frac{-59}{36}}$$

Solve

$$x = \frac{1}{6} \pm \frac{\sqrt{59}}{6}i$$

Practice

a) Given: $\angle C \cong \angle E$
 $\overline{CD} \cong \overline{ED}$

Outline a proof that shows:

$$\triangle BCD \cong \triangle AED$$

-- Mark **givens** and **proves**

-- Mark any obvious conclusions **(in red)**

-- Find way to prove Δ 's

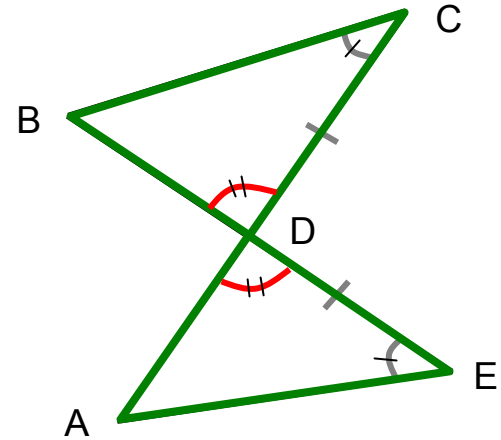
SSS - NO

SAS - NO

ASA - YES

$$\therefore \triangle BCD \cong \triangle AED$$

ASA (AAAS)



b) Simplify: $\frac{2+i-2i^3}{2i^3+4}$

Reduce top and bottom and rearrange

$$= \frac{2+i-2i^2i}{2i^2i+4}$$

$$= \frac{2+i+2i}{-2i+4}$$

$$= \frac{2+3i}{4-2i}$$

Clear bottom with the conjugate $(4+2i)$

$$= \frac{2+3i}{4-2i} \cdot \frac{4+2i}{4+2i}$$

$$= \frac{8 + 4i + 12i + 6i^2}{16 + 8i - 8i - 4i^2}$$

$$= \frac{8 + 16i - 6}{16 + 4}$$

$$= \frac{2 + 16i}{20}$$

$$= \frac{2}{20} + \frac{16}{20}i$$

$$= \frac{1}{10} + \frac{4}{5}i$$

c) Is the following argument valid or invalid?

If the light is on then the switch is on.

The switch is not on.

∴ The light is not on.

Starts out invalid because going backwards on the major premise.

But also doing the negation of the second half, so can't tell.

Do contrapositive to check.

If the switch is not on then the light is not on.

The switch is not on.

∴ The light is not on.

Valid.