Answers to Odd-Numbered Problems and Self-Assessment

Chapter 1

1. a. One.

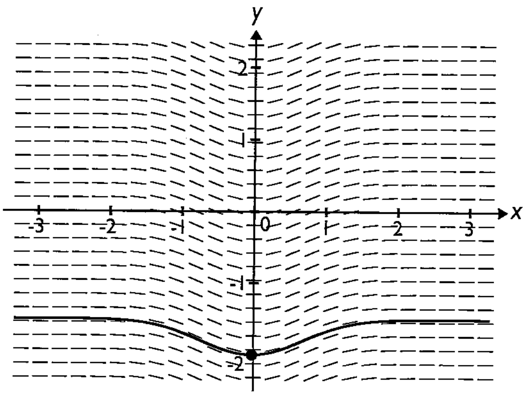

Figure S1.1

b. Figure S1.1 shows the solution through $(0, -2)$. The turning points appear to lie on the y-axis and the solutions to have two points of inflection.

c. The general solution is $y = -\frac{1}{2}e^{-x^2} + C$ and the particular solution through $(0, -2)$ is $y = -\frac{1}{2}e^{-x^2} - \frac{3}{2}$, which has a minimum at $(0, -2)$ and points of inflection at $x = \pm\frac{1}{\sqrt{2}}$.

3. a. One type of solution, all even.

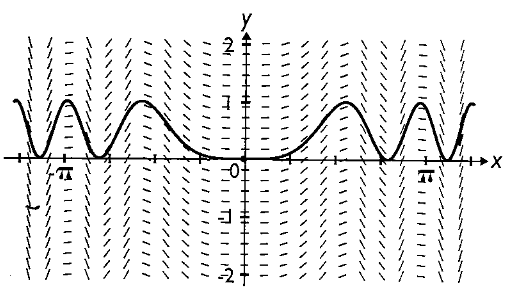

Figure S1.3

b. The general solution is $y = -\frac{1}{2}\cos(x^2) + C$. Figure S1.3 shows the particular solution passing through $(0, 0)$.

5. a. The solution through the origin is odd; all other solutions are neither even nor odd. The solutions appear to have a point of inflection on the y-axis and to increase everywhere.

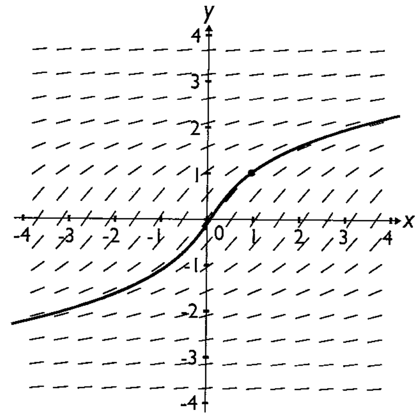

Figure S1.5

b. The general solution is $y = 1.5\tan^{-1}(x) + C$. Figure S1.5 gives the particular solution passing through $(1, 1)$, which does confirm **5a**.

7. a. There appear to be two types of solutions, one on $x < 1$ and one on $x > 1$.

b. There appears to be a vertical asymptote at $x = 1$, which makes sense: $\frac{dy}{dx} = \frac{x^2}{x - x^2}$, so $|y'| \to \pm\infty$ as $x \to 1$.

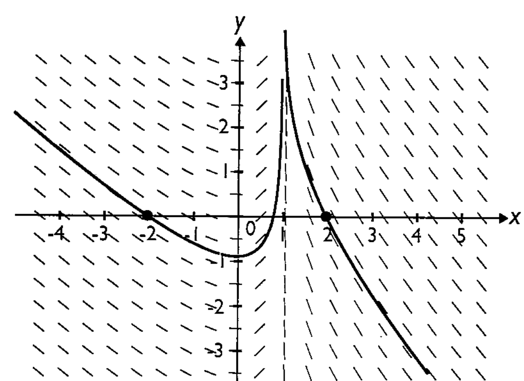

Figure S1.7

c. The general solution is

$$y = -\ln|1 - x| - x + C.$$

The solutions through (2, 0) and (−2, 0) are shown in Figure S1.7 and they do confirm **7a** and **7b**.

9. Figure S1.9 shows three types of solutions. Two are concave down and one is concave up; all have symmetrical finite domains. The solutions shown pass through (0, 4), (0, 2), and (0, −3).

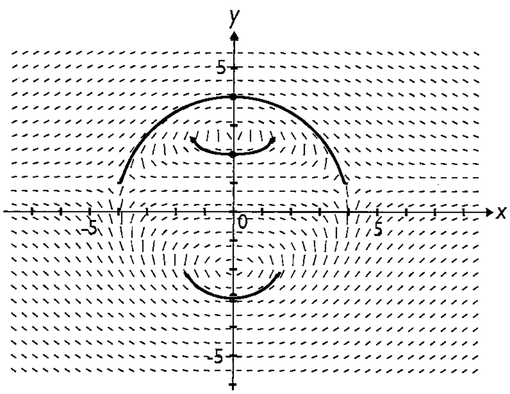

Figure S1.9

11. a. Four types of solutions, one in each quadrant.

b. $(xy)' = C$

Chapter 2

1.

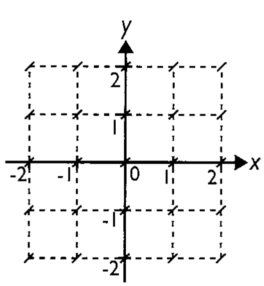

Figure S2.1

3.

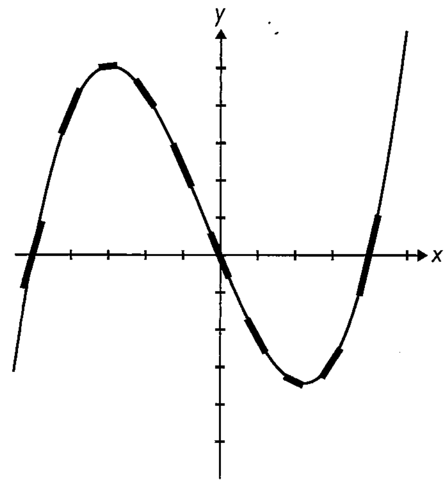

Figure S2.3

5. The slopes should be negative in the second quadrant because $-yx^2 < 0$.

7.

i	ii	iii	iv	v	vi
c	e	a	d	f	b

9.

i	ii	iii	iv	v	vi
a	f	b	e	c	d

11. a. $\frac{1}{2}$

b. Along the diagonal $y = x$ the slopes are undefined, as the tangent lines become steeper and steeper until vertical. Therefore, the derivative does not exist on $y = x$.

13. Because the derivative is a function of y alone, the slopes at each point are independent of x and so the tangent lines are parallel along each horizontal line $y = c$.

15. Because the tangent lines appear to be parallel along each horizontal line $y = c$, the derivative must be independent of x and so is a function of y only.

17. $y' = -1$.

19. The slope fields will be symmetrical through the origin.

Chapter 3

1. a. Figure S3.1 shows a sketch of the solution passing through (0, 2). On that curve, $z(2) \approx 3$.

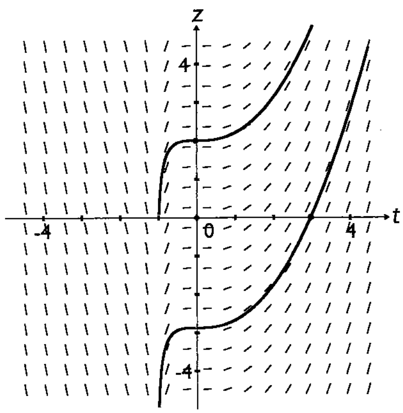

Figure S3.1

b. $z = 0.5t^2 - t + \ln|t + 1| + 2$

and $z(2) \approx 3.099$

3. a. Figure S3.3 shows the solution through $(\pi, 1)$ where $y(\frac{\pi}{2}) \approx 1.25$.

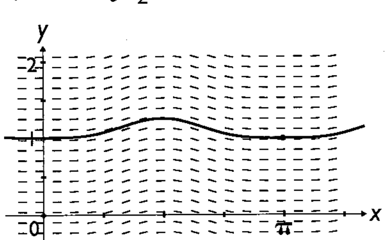

Figure S3.3

b. $y = \frac{1}{4}\sin^4(x) + 1$ and so $y(\frac{\pi}{2}) = \frac{5}{4}$.

5. Figure S3.5 shows the solutions through $(1, -2)$ and through $(1, 3)$.

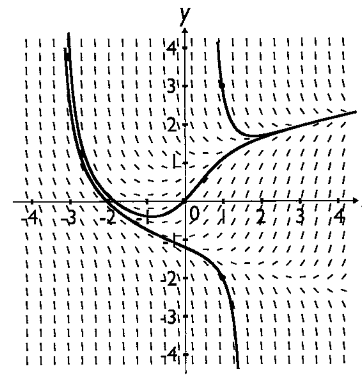

Figure S3.5

a. $y(-2) \approx 0$.

b. $y(3) \approx 2$.

7. a. Substitute $y = -1 - x$ into the differential equation to get $-1 = x + (-1 - x) = -1$.

b. Substitute $y = mx + b$. This gives

$$m = x + (mx + b) = (m + 1)x + b.$$

This equation should hold for all values of x, in particular for $x = 1$. Therefore, $m = m + 1 + b$ so that $b = -1$. The equation should hold for $x = 0$ as well and so $m = b = -1$.

c. If (a, b) lies below $y = -1 - x$ then $b < -1 - a$, so $y' = a + b$ is negative and the solution is decreasing. Therefore, any point on the solution with $x < a$ must have $y > b$ and yet still lie below $y = -1 - x$. Such a point must be closer to the boundary than (a, b).