

**Challenge: Skills and Applications**

For use with pages 463–469

In Exercises 1–3, find the value of  $n$  that makes the question true.

1.  $\left(\frac{x^3}{x^5}\right)^n = x^{-6}$

2.  $\frac{y^{4n-1}}{y^{2n}} = y^5$

3.  $\left(\frac{y^{3n}}{y^{5n-4}}\right)^2 = y^{20}$

In Exercises 4–6, assuming the power of a power property

[ $(a^m)^n = a^{mn}$ ] works for positive integers  $m$  and  $n$ , show that the property is true in each situation.**Example:**  $m$  is a negative integer,  $n$  is a positive integer.**Solution:** Let  $m = -k$ , where  $k$  is a positive integer. Then

$$(a^m)^n = (a^{-k})^n = \left(\frac{1}{a^k}\right)^n = \frac{1^n}{(a^k)^n} = \frac{1}{a^{kn}} = a^{-kn} = a^{mn}$$

4.  $m$  is a positive integer,  $n$  is a negative integer.
5.  $m$  is a negative integer,  $n$  is a negative integer.
6.  $m$  is a positive integer,  $n$  is zero.

In Exercises 7–9, use the following information.

The 17th-century French mathematician Pierre de Fermat proved a theorem about any prime number  $p$  and any integer  $a$  that is not divisible by  $p$ . You can discover this fact for  $p = 5$  and  $a = 2, 3, 4, 6$ , and  $7$  by calculating the values of  $2^4, 3^4, 4^4, 6^4$ , and  $7^4$ .

7. What remainders do you get when you divide your answers by 5?
8. If  $p = 7$ , you can verify Fermat's theorem by calculating  $2^6, 3^6, 4^6, 5^6$ , and  $6^6$ . What remainders do you get when you divide these numbers by 7?
9. Based on your answers from Exercises 7 and 8, state Fermat's theorem. Start your statement with, "If  $p$  is a prime number and  $a$  is not divisible by  $p$ ..."