Classwork: More on graphs of exponential and logarithmic functions

**Exponential graphs revisited**

Shown below are the two most important shapes for graphs of exponential functions \( f(x) = a \cdot b^x \).

| When \( a \) is a positive number and \( b > 1 \),  
| the graph is increasing, and looks like this: |
| When \( a \) is a positive number and \( 0 < b < 1 \),  
| the graph is decreasing, and looks like this: |

Some notes about graphs of \( f(x) = a \cdot b^x \):

- The value of \( b \) determines whether the graph is increasing or decreasing.
- In both cases, the \( y \)-intercept is at the point \((0, a)\).
- In both cases, the graph has the \( x \)-axis as an asymptote (a line to which the graph gets closer and closer, but never actually reaches).

**Problems**

1. Without a calculator, sketch the shape of the graph for each function given below. Label the \( y \)-intercept with its coordinates.

   a. \( f(x) = 3 \cdot 2^x \)

   b. \( f(x) = 4 \cdot 1.25^x \)

   c. \( f(x) = 5 \cdot 0.8^x \)

   d. \( f(x) = 0.9 \cdot \left(\frac{3}{4}\right)^x \)

   e. \( f(x) = 0.9 \cdot \left(\frac{4}{3}\right)^x \)

   f. \( f(x) = 0.6^x \)

2. For all the functions in problem 1, what are the domain and the range?
3. More graphs of exponential and logarithmic functions

a. Complete the input-output tables for the functions \( f(x) = \log_{\frac{1}{2}} x \) and \( g(x) = \left(\frac{1}{2}\right)^x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( x )</th>
<th>( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/16</td>
<td>4</td>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>1/8</td>
<td></td>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>1/4</td>
<td></td>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>1/2</td>
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<td>0</td>
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<tr>
<td>1</td>
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<td>2</td>
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<td>2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

b. Graph both functions on the grid below. Also, draw the graph of the line \( y = x \).

c. What can you say about the graphs of \( f(x) \) and \( g(x) \)?
Connection: inverse functions

This problem connects exponents and logs to something we studied previously, the idea of functions having inverses.

4. Find the inverse \( f^{-1}(x) \) of each function \( f(x) \) below. Two examples are shown.
Remember that the key steps are “Change” (\( x \)’s to \( y \)’s, \( y \)’s to \( x \)’s) and “Solve” (for \( y \)).

**Example:** Find the inverse of \( f(x) = x^4 \)

Start with:
\[
y = x^4
\]
Change:
\[
x = y^4
\]
Solve:
\[
x^{1/4} = (y^4)^{1/4}
\]
\[
x^{1/4} = y
\]
Answer: \( f^{-1}(x) = x^{1/4} \)

**Example:** Find the inverse of \( f(x) = 4^x \)

Start with:
\[
y = 4^x
\]
Change:
\[
x = 4^y
\]
Solve:
\[
\log_4 x = y \quad \text{(used the “shapes” rewriting)}
\]
Answer: \( f^{-1}(x) = \log_4 x \)

a. Find the inverse of \( f(x) = x^3 \).

b. Find the inverse of \( f(x) = 3x^5 \). **Hint:** First step in the “Solve” phase is dividing by 3.

c. Find the inverse of \( f(x) = 1.06^x \).

d. Find the inverse of \( f(x) = 5000 \cdot 1.06^x \).
HOMEWORK PROBLEMS (Begin review for quiz on Dec. 18):

1. Answer these questions about the function \( f(x) = 1.3 \cdot (0.2)^x \).
   a. Is this a growth function (increasing) or a decay function (decreasing)? Explain how you can tell just by looking at the formula.

   b. Suppose this function formula came from a word problem involving a percent increase or decrease. Which is it (increase or decrease), and what would the percentage be?

   c. Which of these is the shape of the graph of \( f(x) \)?

   d. Find these function values.

      \[
      f(2) = \, \\
      f(1) = \, \\
      f(0) = \, \\
      f(-1) = \,
      \]

   e. Solve the equation \( f(x) = 1 \) algebraically (not graphically) using a calculator.

   f. Solve the equation \( f(x) = 3 \) algebraically (not graphically) using a calculator.
Answer questions 2 and 3 without using your calculator.

2. \( f(x) = \log_5(x + 5) - 2 \)
   
   a. Find the \( y \) – intercept.
   
   b. Find the zero.

   c. Evaluate \( f(-4) \)

   d. Solve \( f(x) = 1 \)

3. Let \( f(x) = 3 \cdot 2^x - 12 \).
   
   a. Find the \( y \) – intercept.
b. Find the zero.

c. Evaluate $f(4)$

d. Solve $f(x) = -10.5$

4. Given each function formula, find the specified function value. Get the simplest answer you can without using a calculator.

a. $f(x) = -6(x - 2) + 5$. Find $f(0)$.

b. $f(x) = 5 \cdot 2^x$. Find $f(-3)$.

c. $f(x) = \frac{3}{x}$. Find $f(\frac{1}{3})$. 
d. \( f(x) = x^{-3} \). Find \( f(\frac{1}{2}) \).

e. \( f(x) = \log_a(x) \). Find \( f(4^{10}) \).

f. \( f(x) = \log_{\frac{1}{6}}(x) \). Find \( f(36) \).

g. \( f(x) = x^3 \). Find \( f(-4) \).

h. \( f(x) = x^3 \). Find \( f(x^2) \). \textbf{Hint:} Put \( x^2 \) in the place of \( x \) in the function formula.

5. Rewrite each equation according to the instructions below.

a. Rewrite as a logarithmic equation: \( 5^{-3} = \frac{1}{125} \)

b. Rewrite as an exponential equation: \( \log_4(16) = 2 \)

c. Rewrite in slope-intercept form: \( f(x) = -20(x - 10) - 50 \)
d. Rewrite as a logarithmic equation: \(10^x = 30\)

e. Rewrite as an exponential equation: \(\log_2 x = 6\)

f. Rewrite as an exponential equation: \(3 \log_2 x = 6\)

g. Rewrite without any negative exponents: \(y = x^{-5}\)

h. Rewrite without any parentheses: \(y = (4x)^3\)