In math, the word “sequence” is used in much the same way as in ordinary English. When we say a collection of objects or events is in sequence, we usually mean that collection is ordered so there is an identified first member, second member, and so on.

**Sequence**

An **infinite sequence** is a function whose domain is the set of positive integers. The function values

\[ a_1, a_2, a_3, a_4, \ldots, a_n, \ldots \]

are the **terms** of the sequence. If the domain of a function consists of the first \( n \) positive integers only, the sequence is a **finite** sequence.
Write the first four terms of each sequence:

(a) \( a_n = 4n + 1 \)  
(b) \( b_n = 3(2)^{n-1} \)  
(c) \( c_n = 5 + (-1)^{n+1} \)

\[ a_1 = 5 \quad b_1 = 3 \quad c_1 = 6 \]
\[ a_2 = 9 \quad b_2 = 6 \quad c_2 = 4 \]
\[ a_3 = 13 \quad b_3 = 12 \quad c_3 = 6 \]
\[ a_4 = 17 \quad b_4 = 24 \quad c_4 = 4 \]
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Write a rule that finds the apparent $n$th term for the given sequence:

(a) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots$

\[
\begin{align*}
a_1 &= \frac{1}{3} = \frac{1}{3^1} \\
a_2 &= \frac{1}{9} = \frac{1}{3^2} \\
a_3 &= \frac{1}{27} = \frac{1}{3^3} \\
a_4 &= \frac{1}{81} = \frac{1}{3^4} \\
& \vdots \\
A_n &= \frac{1}{3^n}
\end{align*}
\]

(b) $2, 6, 12, 20, \ldots$

\[
\begin{align*}
a_n &= an^2 + bn + c \\
a_1 &= a + b + c = 2 \\
a_2 &= 4a + 2b + c = 6 \\
a_3 &= 9a + 3b + c = 12
\end{align*}
\]

\[
\begin{align*}
2a &= 2 \\
a &= 1 \\
3b &= 4 \\
b &= 1 \\
1 + 1 + c &= 2 \\
c &= 0
\end{align*}
\]
**Series**

When the terms in a sequence are added, the resulting expression is a **series**. A **series** can be **infinite** of **finite**.

**Finite sequence**
3,6,9,12,15

**Infinite sequence**
3,6,9,12,15,...

**Finite series**
3+6+9+12+15

**Infinite series**
3+6+9+12+15+...

We can use **summation notation** to write a series. For example, the finite series 3+6+9+12+15 can be written in summation notation as \( \sum_{i=1}^{5} 3i \).

- \( i \) is called the **index of summation**
- 1 is the **lower limit**
- 5 is the **upper limit**

The summation is read as “the sum from \( i \) equals 1 to 5 of \( 3i \)”. Summation notation is also called **sigma notation** because it uses the uppercase Greek letter sigma.
Write each series with summation notation:

(a) $5 + 10 + 15 + \ldots + 100$

(b) $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \ldots$

\[ a) \sum_{i=1}^{20} 5i \quad b) \sum_{i=1}^{\infty} \frac{i}{i+1} \]
Find the sum of the series:

(a) \[\sum_{i=1}^{4} 2i = 2 \sum_{i=1}^{4} i = 2(1+2+3+4) = 2(10) = 20\]

(b) \[\sum_{n=4}^{6} (2+n)^2 = (2+4)^2 + (2+5)^2 + (2+6)^2
= 6^2 + 7^2 + 8^2 = 36 + 49 + 64 = 149\]
11.1 – DAY 1 Practice

Write the first six terms of the sequence

1. \( a_n = \frac{n+4}{n} \)
   
   \[ a_1 = 5 \quad a_5 = \frac{9}{5} \]
   \[ a_2 = 3 \quad a_6 = \frac{10}{6} = \frac{5}{3} \]
   \[ a_3 = \frac{7}{3} \]
   \[ a_4 = 1 \]

2. \( b_n = (-1)^n (n+1) \)
   
   \[ b_1 = -2 \quad b_5 = -6 \]
   \[ b_2 = 3 \quad b_6 = 7 \]
   \[ b_3 = -4 \]
   \[ b_4 = 5 \]

Write the next term in the sequence. Then write a rule for the apparent nth term of the sequence.

3. \( \frac{1}{2}, 1, \frac{3}{2}, 2, ..., \)
   
   \[ \frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \frac{5}{2}, ..., \]
   \[ a_n = \frac{n}{2} \]

4. \( 4, 3, 2, 1, ..., \)
   
   \[ 4, 3, 2, 1, 0, ..., \]
   \[ a_n = 5 - n \]

5. \( 4, 9, 16, 25, ..., \)
   
   \[ 4, 9, 16, 25, 36, ..., \]
   \[ a_n = (n+1)^2 \]

Write the series with summation notation.

6. \[ 1 + 2 + 4 + 8 + 16 + 32 \]
   
   \[ \sum_{i=1}^{6} 2^{i-1} \]
   
   or
   
   \[ \sum_{i=0}^{5} 2^i \]

7. \[ \frac{1}{4} - \frac{1}{9} + \frac{1}{16} - \frac{1}{25} + \frac{1}{36} + ... \]
   
   \[ \sum_{i=1}^{\infty} (-1)^{i+1} \frac{1}{(i+1)^2} \]
   
   or
   
   \[ \sum_{i=0}^{\infty} (-1)^i \frac{1}{(i+2)^2} \]
Find the sum of the series.

8. \[ \sum_{n=2}^{5} \frac{2n}{2(n-1)} \]

\[ \sum_{n=2}^{5} \frac{n}{n-1} \]

\[ \frac{2 + \frac{3}{2} + \frac{4}{3} + \frac{5}{4}}{24 + 18 + 16 + 15} \]

\[ \frac{24 + 18 + 16 + 15}{12} \]

\[ \frac{73}{12} \]

9. \[ \sum_{i=0}^{3} \left( \frac{1}{2} \right)^i \]

\[ 6 + \frac{6}{2} + \frac{6}{4} + \frac{6}{8} \]

\[ 6 + 3 + \frac{3}{2} + \frac{3}{4} \]

\[ \frac{36 + 6 + 3}{4} \]

\[ \frac{45}{4} \]

10. \[ \sum_{n=5}^{7} (2n - 1) \]

\[ 5(9) + 6(11) + 7(13) \]

\[ 45 + 66 + 91 \]

\[ 202 \]
In an arithmetic sequence, the difference between consecutive terms is constant. The constant difference is called the common difference and is often denoted by \( d \).

Decide whether the sequence is arithmetic:

(a) \(-3, 1, 5, 9, 13, \ldots\)  

\[
\begin{align*}
\begin{array}{c}
+4 \\
+4 \\
+9 \\
+4
\end{array}
\end{align*}
\]

\( a_1 = -3 \)  
\( d = 4 \)

Yes

(b) \(2, 5, 10, 17, 26, \ldots\)  

\[
\begin{align*}
\begin{array}{c}
+3 \\
+7
\end{array}
\end{align*}
\]

No

(c) \(18, 16, 14, 12, 10, \ldots\)  

\[
\begin{align*}
\begin{array}{c}
-2 \\
-2 \\
-2 \\
-2
\end{array}
\end{align*}
\]

\( a_1 = 18 \)  
\( d = -2 \)

Yes
The nth term of an arithmetic sequence with common difference $d$ is given by:

$$a_n = dn + c$$

where $c = a_1 - d$

**Example:**

Write a rule for the $n$th term of the sequence: 50, 44, 38, 32, …. Then find $a_{20}$.

$$a_1 = 50 \quad d = -6$$

$$a_n = 50 + (n-1)(-6)$$

$$a_n = 50 - 6n + 6$$

$$a_n = 56 - 6n$$

$$a_{20} = 56 - 6(20) = 56 - 120$$

$$a_{20} = -64$$
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**Example:**

Write a rule for the $n$th term of the sequence:
8, 19, 30, 41, …

\[
\begin{align*}
A_1 &= 8 \\
\text{d} &= 11
\end{align*}
\]

\[
A_n = 8 + (n-1)(11)
\]

\[
A_n = 8 + 11n - 11 = -3 + 11n
\]
Example:

The 8th term of an arithmetic sequence is 25 and the 12th term is 41. Write the first five terms of this sequence.

\[ a_8 = 25 \quad \Rightarrow \quad 25 = a_1 + 7d \quad \Rightarrow \quad 16 = 4d \Rightarrow d = 4 \]

\[ a_{12} = 41 \quad \Rightarrow \quad 41 = a_1 + 11d \]

\[ 25 = a_1 + 28 \Rightarrow a_1 = -3 \]

\[ a_n = -3 + (n-1)(4) = -3 + 4n - 4 \]

\[ a_n = -7 + 4n \]

\[ a_1 = -3 \]
\[ a_2 = 1 \]
\[ a_3 = 5 \]
\[ a_4 = 9 \]
\[ a_5 = 13 \]
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**Example:**

Find the arithmetic mean of 8 and 24.

\[
\begin{align*}
A_1 &= 8 \\
A_2 &= ? \\
A_3 &= 24
\end{align*}
\]

\[
\begin{align*}
8 + d &= 24 - d \\
2d &= 24 - 8 \\
2d &= 16 \\
d &= 8
\end{align*}
\]

\[A_2 = 16\]
Example:

Insert two arithmetic means between 16 and 60

\[ A_1 = 16 \]
\[ A_2 = 16 + d \]
\[ A_3 = 16 + 2d \]
\[ A_4 = 16 + 3d \]

\[ 16 + 3d = 60 \]
\[ 3d = 44 \]
\[ d = \frac{44}{3} \]

\[ A_2 = 16 + \frac{44}{3} = \frac{52 + 44}{3} = \frac{96}{3} = 32 \]

\[ A_3 = \frac{96}{3} + \frac{44}{3} = \frac{140}{3} \]
The expression formed by adding the terms of an arithmetic sequence is called an arithmetic series. The sum of the first n terms of an arithmetic series is denoted by $S_n$.

Let’s derive the formula for the sum of a finite arithmetic series:

$$S_n = a_1 + a_2 + a_3 + \ldots + a_n$$
$$S_n = a_1 + a_1 + d + a_1 + 2d + a_1 + 3d + \ldots + a_1 + (n-1)d$$
$$S_n = n a_1 + d [1+2+3+\ldots+n-1]$$
$$S_n = n a_1 + d \left( \frac{n(n+1)}{2} - 1 \right)$$
$$S_n = n a_1 + n \frac{a_1}{2} + d \left( \frac{(n-1)n}{2} \right)$$
$$S_n = \frac{n}{2} \left( a_1 + a_n \right)$$

The Sum of a Finite Arithmetic Series

$$S_n = \frac{n}{2} (a_1 + a_n)$$
Example:

Given the arithmetic sequence 4, 7, 10, 13, 16, 19, ...

(a) find the sum of the first 30 terms in the sequence

(b) Find \( n \) such that \( S_n = 175 \)

\[
\begin{align*}
\text{(a) } & A_1 = 4 \\
& d = 3 \\n& A_n = 4 + 3(n - 1) = 1 + 3n \\
& S_{30} = \frac{30}{2} \left( 4 + 95 \right) \\
& S_{30} = 15(99) \\
& S_{30} = 1,485 \\
\end{align*}
\]

\[
\begin{align*}
\text{(b) } & S_n = \frac{n}{2} \left( 4 + A_n \right) \\
& 175 = \frac{n}{2} \left( 4 + 1 + 3n \right) \\
& 350 = n(5 + 3n) \\
& 350 = 5n + 3n^2 \\
& 3n^2 + 5n - 350 = 0 \\
& 3n^2 + 35n - 30n - 350 = 0 \\
& n(3n + 35) - 10(3n + 35) = 0 \\
& (3n + 35)(n - 10) = 0 \\
& n = -\frac{35}{3} \quad n = 10
\end{align*}
\]
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Example:

The first row in a theater has 22 seats, and each row after the first has 3 more seats than the row before it. There are 32 rows of seats.

(a) How many seats are in the theater?
(b) Thirty-nine students from a class want to sit in the same row. How close to the front can they sit?

\[ a_1 = 22 \]
\[ d = 3 \]
\[ n = 32 \]

\[ a_{32} = 22 + 31 \cdot 3 = 22 + 93 = 115 \]

\[ S_{32} = \frac{32}{2} (22 + 115) = 16 (137) = 2,192 \text{ seats} \]

\[ a_n \geq 39 \]
\[ 22 + 3(n-1) \geq 39 \]
\[ 19 + 3n \geq 39 \]
\[ 3n \geq 20 \]
\[ n \geq \frac{20}{3} \]

The closest row will be 7th.
Write a rule for the nth term of the arithmetic sequence. Then find $a_{20}$.

1. 1, 7, 13, 19, 25, …
   \[ a_1 = 1, \quad d = 6 \]
   \[ a_n = 6n - 5 \]
   \[ a_{20} = 115 \]

2. 2.5, 2.2, 1.9, 1.6, …
   \[ a_1 = 2.5, \quad d = -0.3 \]
   \[ a_n = 2.8 - 0.3n \]
   \[ a_{20} = -3.2 \]

3. \( \frac{9}{4}, \frac{10}{4}, \frac{11}{4}, \frac{12}{4}, ..., \)
   \[ a_1 = \frac{9}{4}, \quad d = \frac{1}{4} \]
   \[ a_n = \frac{9}{4} + \frac{1}{4}(n-1) \]
   \[ a_{20} = 7 \]

4. $d = 0.2, \quad a_1 = 16$
   \[ a_n = 16 + 0.2(n-1) \]
   \[ a_n = 16 + 0.2n - 0.2 \]
   \[ a_n = 15.8 + 0.2n \]

5. $a_6 = 27.2, \quad a_{13} = 44$
   \[ a_{13} = a_1 + 12d = 44 \]
   \[ a_6 = a_1 + 5d = 27.2 \]
   \[ 7d = 16.8 \]
   \[ d = 2.4 \]
   \[ a_1 = 15.2 \]
   \[ a_n = 15.2 + 2.4(n-1) \]
   \[ a_n = 12.8 + 2.4n \]

6. $a_{15} = -19, \quad a_{24} = -16$
   \[ a_{24} = a_1 + 23d = -16 \]
   \[ a_{15} = a_1 + 14d = -19 \]
   \[ 9d = 3 \]
   \[ d = \frac{1}{3} \]
   \[ a_1 = -7\frac{1}{3} \]
   \[ a_n = -7\frac{1}{3} + \frac{1}{3}(n-1) \]
   \[ a_n = -24 + \frac{1}{3} \]
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For part (a), find the sum of the first n terms of the arithmetic series. For part (b), find n for the given sum $S_n$.

7. $40 + 37 + 34 + 31 + \ldots$  
   (a) $n = 20$  
   (b) $S_n = -208$

8. $-6 + (-2) + 2 + 6 + \ldots$  
   (a) $n = 28$  
   (b) $S_n = 442$

7. a) $A_1 = 40 \quad d = -3$  
   $A_2 = 40 - 3(n - 1)$  
   $A_{20} = -17$  
   $S_{20} = \frac{20}{2} (40 - 17)$  
   $S_{20} = 230$

   b) $S_n = \frac{n}{2} (40 + 43 - 3n)$
   
   $-208 = \frac{n}{2} (83 - 3n)$
   
   $-416 = -3n^2 + 83n$

   $3n^2 - 83n - 416 = 0$
   
   $n = 32$  
   ← use quadratic formula

8. a) $A_1 = -6 \quad d = 4$  
   $A_n = -6 + 4(n - 1)$  
   $A_{28} = 102$

   $S_{28} = \frac{28}{2} (-6 + 102) = 1,344$

   $S_{28} = 1,344$
9. A concert hall has 42 rows of seats. The first row has 15 seats and each row after the first has 4 more than the previous row. Seats in rows 1-11 cost $24, seats in rows 12-28 cost $18, and seats in rows 29-42 cost $12. How much does the concert hall take in for a sold-out event?

\[
\begin{align*}
A_1 &= 15 \\
A_n &= 15 + 4(n-1) \\
A_{42} &= 11 + 4(42) = 11 + 168 = 179 \\
A_{11} &= 11 + 4(11) = 11 + 44 = 55 \\
A_{28} &= 11 + 4(28) = 11 + 112 = 123 \\
S_{42} &= \frac{42}{2}(15 + 179) = 4107 \\
S_{28} &= \frac{28}{2}(15 + 123) = 1932 \\
S_{11} &= \frac{11}{2}(15 + 55) = 385
\end{align*}
\]

- \# of seats row 29-42: $4107 - 1932 = 2175$
- \# of seats row 12-28: $1932 - 385 = 1547$
- \# of seats row 1-11: 385

\[24(385) + 18(1547) + 12(2175) = \$62,790\]
10. The 3rd term of an arithmetic sequence is -10 and the 8th term is -45. Find the 20th term in the sequence.

\[
\begin{align*}
A_8 &= -45 = a_1 + 7d \\
A_3 &= -10 = a_1 + 2d \\
-35 &= 5d
\end{align*}
\]

\[d = -7 \Rightarrow a_1 - 14 = -10 \Rightarrow a_1 = 4\]

\[a_n = 4 - 7(n-1)\]

\[a_{20} = 11 - 7(20) = -129\]

\[a_{20} = -129\]
11. In an arithmetic sequence, \( S_n = 210 \), \( a_n = 24 \), and \( n = 14 \). Find \( a_1 \).

\[
\begin{align*}
210 &= \frac{14}{2} (a_1 + 24) \\
210 &= 7(a_1 + 24) \\
30 &= a_1 + 24 \\
\Rightarrow a_1 &= 6
\end{align*}
\]
12. A certain finite arithmetic sequence has a common difference of 5 and a total of 14 terms. \( S_{14} = 651 \). Find \( a_1 \) and \( a_{14} \).
13. Find $x$ so the numbers $4x-1$, $2x+2$, $2x-3$ are the terms of an arithmetic sequence. Find the sum of the first 30 terms.

\[
\begin{align*}
2x+2 &= 4x-1 + d \\
3 - 2x &= d \\
2x+2 + 3 - 2x &= 2x - 3 \\
5 &= 2x - 3 \\
8 &= 2x \\
4 &= x
\end{align*}
\]

\[
\begin{align*}
A_1 &= 15 \\
d &= -5 \\
A_n &= 15 - 5(n-1) \\
A_n &= 20 - 5n \\
A_{30} &= -130
\end{align*}
\]

\[
S_{30} = \frac{30}{2} \left( 15 - 130 \right) = -1,725
\]
14. There are three numbers in the ratio 3:4:7. If four is added to the middle number, that resulting number will be the second term in an arithmetic sequence of which the other two numbers are the first and third terms. Find the three numbers.

Numbers are: $3x$, $4x$, and $7x$

\[
\begin{align*}
A_1 &= 3x \\
A_2 &= 4x + 4 \\
A_3 &= 7x
\end{align*}
\]

\[
\begin{align*}
3x + d &= 4x + 4 \\
d &= x + 4
\end{align*}
\]

\[
\begin{align*}
4x + 4 + x + 4 &= 7x \\
5x + 8 &= 7x \\
8 &= 2x \\
x &= 4
\end{align*}
\]

Numbers: 12, 16, and 28
In a geometric sequence, the ratio of any term to the previous term is constant. This constant is called the common ratio and is denoted by \( r \).

**Example:**

Decide whether the sequence is geometric.

(a) \(1, 2, 6, 24, 120, \ldots\) \[ \text{No} \]

\[ \frac{2}{1} = 2, \quad \frac{6}{2} = 3, \quad \frac{24}{6} = 4, \quad \frac{120}{24} = 5 \]

(b) \(81, 27, 9, 3, 1, \ldots\) \[ \text{Yes} \]

\[ r = \frac{27}{81} = \frac{1}{3} \]

(c) \(-4, 8, -16, 32, -64, \ldots\) \[ \text{Yes} \]

\[ r = \frac{-8}{4} = -2 \]
Example:

Find a formula for the nth term of the geometric sequence 6,\(-2,\frac{2}{3}\),… What is the tenth term?

\[
\begin{align*}
A_1 &= 6 \\
A_2 &= -2 \\
r &= \frac{-2}{3}
\end{align*}
\]

Rule for a Geometric Sequence

The nth term of a geometric sequence with first term \(a_1\) and common ratio \(r\) is given by:

\[
a_n = a_1 r^{n-1}
\]
Example:

The 3rd term of a geometric sequence is \( \frac{16}{3} \) and the 7th term of the sequence is \( \frac{256}{243} \). Find the 9th term in the sequence.

\[
\begin{align*}
A_3 &= A_1 r^2 = \frac{16}{3} \\
A_7 &= A_1 r^6 = \frac{256}{243} \\
\Rightarrow & \quad r^4 = 16 \Rightarrow r = \pm \frac{2}{3}
\end{align*}
\]

\[
\begin{align*}
A_1 \left( \pm \frac{2}{3} \right)^2 &= \frac{16}{3} \\
A_1 \left( \frac{2}{3} \right)^4 &= \frac{16}{3} \\
A_1 &= 1.2
\end{align*}
\]

\[
A_9 = 1.2 \left( \pm \frac{2}{3} \right)^8 = \frac{1024}{2187}
\]
Example: You may remember this concept from Geometry.

Find the geometric mean of 10 and 40.

\[
10, 10r, \frac{10r^2}{40} \quad r^2 = 4 \\
\quad r = \pm 2
\]

Geometric Mean: 20 or -20
Example:
Insert three geometric means between 7 and 567.

\[ 7, \, 7r, \, 7r^2, \, 7r^3, \, 567 \]

\[ 7r^4 = 567 \]
\[ r^4 = 81 \Rightarrow r = \pm 3 \]

If \( r = 3 \): 21, 63, 189
If \( r = -3 \): -21, 63, -189
The expression formed by adding the terms of a geometric sequence is called a **geometric series**. As with an arithmetic series, the sum of the first \( n \) terms of a geometric series is denoted by \( S_n \)

Let’s derive the formula for the sum of a finite geometric series:

\[
S_n = a_1 + a_2 + a_3 + \ldots + a_n
\]

\[
S_n = a_1 + a_1r + a_1r^2 + a_1r^3 + \ldots + a_1r^{n-1}
\]

\[
S_n = a_1 \left( 1 + r + r^2 + r^3 + \ldots + r^{n-1} \right)
\]

\[
S_n = a_1 \left( \frac{1 - r^n}{1 - r} \right)
\]

**The Sum of a Finite Geometric Series**

\[
S_n = \frac{a_1 \left( 1 - r^n \right)}{1 - r}
\]
Example:

Find the sum of the first 10 terms in the geometric series given by $1 + 5 + 25 + 125 + \ldots$

$$S_{10} = \frac{a_1(1-r^{10})}{1-r} = \frac{1(1-5^{10})}{1-5} = 2,441,406$$
Example:

Find $n$ such that $S_n = 3906$ using the geometric series from the previous example.

\[
\begin{align*}
    a_1 &= 1 \\
r &= 5
\end{align*}
\]

\[
3906 = 1 \left( \frac{1 - 5^n}{1 - 5} \right)
\]

\[
3906(-4) = 1 - 5^n
\]

\[
3906(4) = 5^n - 1
\]

\[
5^n = 15625
\]

\[
h = \log_5 15625
\]

\[
h = 6
\]
Write a rule for the nth term of the geometric sequence. Then find a₈.

1. \( \frac{7}{4}, \frac{21}{16}, \frac{63}{64}, \ldots \)
   - \( a_1 = 7 \)
   - \( r = \frac{3}{4} \)
   - \( a_8 = 7 \left( \frac{3}{4} \right)^7 \)
   - \( a_8 = \frac{15309}{16384} \)

2. 7, 28, 112, 448, …
   - \( a_1 = 7 \)
   - \( r = 4 \)
   - \( a_8 = 7 \cdot 4^7 \)
   - \( a_8 = 114,688 \)

3. \(-\frac{3}{4}, -\frac{3}{16}, \frac{3}{64}, \ldots \)
   - \( a_1 = -3 \)
   - \( r = \frac{1}{4} \)
   - \( a_8 = -3 \cdot \left( \frac{1}{4} \right)^7 \)
   - \( a_8 = -\frac{3}{16384} \)

Write a rule for the nth term of the geometric sequence.

4. \( r = 1.1, a_1 = 5 \)
   - \( a_n = 5 \cdot 1.1^{n-1} \)

5. \( a_3 = -64, a_7 = -\frac{1}{4} \)
   - \( \frac{a_7}{a_3} = \frac{a_1 r^6}{a_1 r^2} = \frac{-\frac{1}{4}}{-64} \)
   - \( r^4 = \frac{1}{256} \)
   - \( r = \pm \frac{1}{4} \)
   - \( a_1 \left( \pm \frac{1}{4} \right)^2 = -64 \)
   - \( a_1 = -1024 \)
   - \( a_n = -1024 \left( \frac{1}{4} \right)^{n-1} \) or \( a_n = -1024 \left( -\frac{1}{4} \right)^{n-1} \)

6. \( a_8 = \frac{1}{9}, a_{15} = 243 \)
   - \( \frac{a_{15}}{a_8} = \frac{a_1 r^{14}}{a_1 r^3} = \frac{243}{\frac{1}{9}} \)
   - \( r^3 = 2187 \)
   - \( r = 3 \)
   - \( a_1 (3)^2 = \frac{1}{9} \)
   - \( a_1 = \frac{1}{19683} \)
   - \( a_n = \frac{3^{n-1}}{19683} \)
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For part (a), find the sum of the first n terms of the geometric series. For part (b), find n for the given sum $S_n$

7. $1 + (-4) + 16 + (-64) + \ldots$ (a) $n = 20$  (b) $S_n = -819$

8. $\frac{9}{2} + \frac{2}{9} + \frac{4}{81} + \ldots$ (a) $n = 20$  (b) $S_n = 5.5$

7. a) $S_{20} = \frac{1}{1 - (-4)} \left(1 - (-4)^{20}\right) = \frac{1 - 4^{20}}{5} = -2.199 \times 10^n$

b) $-819 = \frac{1}{1 - (-4)} \Rightarrow -4095 = 1 - (-4)^n$

$(-4)^n = 1 + 4095$

$(-4)^n = 4096$

$n = \log_4 4096 \Rightarrow n = 6$

8. a) $S_{20} = \frac{9}{2} \left(1 - \left(\frac{2}{9}\right)^{20}\right)$

$S_{20} = \frac{9}{2} \left(\frac{9^{20} - 2^{20}}{9^{20}}\right) = 81/4$

$S_{20} = 81/4$

b) $5.5 = \frac{9}{2} \left(1 - \left(\frac{2}{9}\right)^n\right)$

$\frac{77}{81} = 1 - \left(\frac{2}{9}\right)^n \Rightarrow \left(\frac{2}{9}\right)^n = 1 - \frac{77}{81} = \frac{4}{81} \Rightarrow n = \log_{\frac{2}{9}} \left(\frac{4}{81}\right) = 2$

$n = 2$
9. Find the sum of the series:

(a) \[ \sum_{i=1}^{8} (-3)^{i-1} \]

\[ S_8 = a_1 \left( \frac{1 - r^8}{1 - r} \right) \]

\[ S_8 = 1 \left( \frac{1 - (-3)^8}{1 - (-3)} \right) \]

\[ S_8 = - \frac{6560}{4} = -1640 \]

(b) \[ \sum_{n=1}^{10} -2 \left( \frac{3}{2} \right)^{n-1} \]

\[ S_{10} = -2 \left( \frac{1 - \left(\frac{3}{2}\right)^{10}}{1 - \frac{3}{2}} \right) \]

\[ S_{10} = - \frac{58025}{256} \]
10. Find the first term of a geometric sequence of five terms in which the sum of the five terms is 363 and the common ratio is 3.

\[ a_1 + a_1 \cdot 3 + a_1 \cdot 9 + a_1 \cdot 27 + a_1 \cdot 81 = 363 \]

\[ 121a_1 = 363 \]

\[ a_1 = 3 \]
11. Find the common ratio of a geometric sequence in which \( a_1 = 25 \), \( a_n = 400 \), and \( S_n = 775 \).

\[
\begin{align*}
\alpha_n &= 25 \left( r^{n-1} \right) = 400 \\
    r^{n-1} &= 16
\end{align*}
\]

\[
\begin{align*}
775 &= 25 \left( \frac{1 - r^n}{1 - r} \right) \\
31 &= \frac{1 - r^n}{1 - r} \\
31 (1 - r) &= r \left( \frac{1}{r} - r^{n-1} \right) \\
31 (1 - r) &= r \left( \frac{1}{r} - 16 \right) \\
31 - 31r &= 1 - 16r \\
30 &= 15r \Rightarrow r = 2
\end{align*}
\]
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12. Insert two geometric means between 6 and 384.

\[
6, 6r, 6r^2, 384 \left\{ \begin{array}{l}
6r^3 = 384 \\
r^2 = 64 \\
r = 4
\end{array} \right.
\]

\[
24, 96
\]
13. There are three numbers such that the second is two more than the first and the third is nine times the first. The numbers form a geometric sequence. Find the numbers.

\[
\begin{align*}
\frac{x}{x+2} \quad \frac{9x}{x \neq 0} \\
3x &= x+2 \\
2x &= 2 \\
x &= 1 \\
\frac{r}{x} &= 9x \\
r^2 &= 9 \\
r &= \pm 3 \\
&= 1, 3, 9 \text{ or } -\frac{1}{2}, \frac{3}{2}, -\frac{9}{2}
\end{align*}
\]