# 1.4 Part B

$f(x) = 3x^2 + 5$ $\quad g(x) = x - 4$

$(f \circ g)(x) = f(g(x))$

insert g inside f

inside

$(f \circ g)(x) = 3(x-4)^2 + 5$

$(f \circ g)(x) = h(x)$

suppose $h(x) = (f \circ g)(x)$

$h(x) = f(g(x))$, decompose (split-up) $h(x)$ into $f(x)$ and $g(x)$.

$$h(x) = 1 + \left| \frac{x-1}{2} \right|$$

$$g(x) = \frac{x-1}{2}$$

$$f(x) = 1 + |x|$$

Ex $h(x) = \sin(2x+7) - 1$

$h(x) = f(g(x))$

$g(x) = 2x+7$

$f(x) = \sin(x) - 1$

$$h(x) = e^{(5x)} + \cos(5x)$$

$$g(x) = 5x$$

$$f(x) = e^{x} + \cos(x)$$

# Inverse relations and inverse functions

inverses as "opposites"

addition and subtraction

mult. and div.

$\sqrt{x}$ $x^2$

roots powers

inverses cancel-out each other

Def. of inverse relation: switch x and y values (input with output)

$y = 2x$ $\qquad\qquad$ $y = \frac{x}{2}$

$(5, 10)$ $\qquad\qquad$ $(10, 5)$

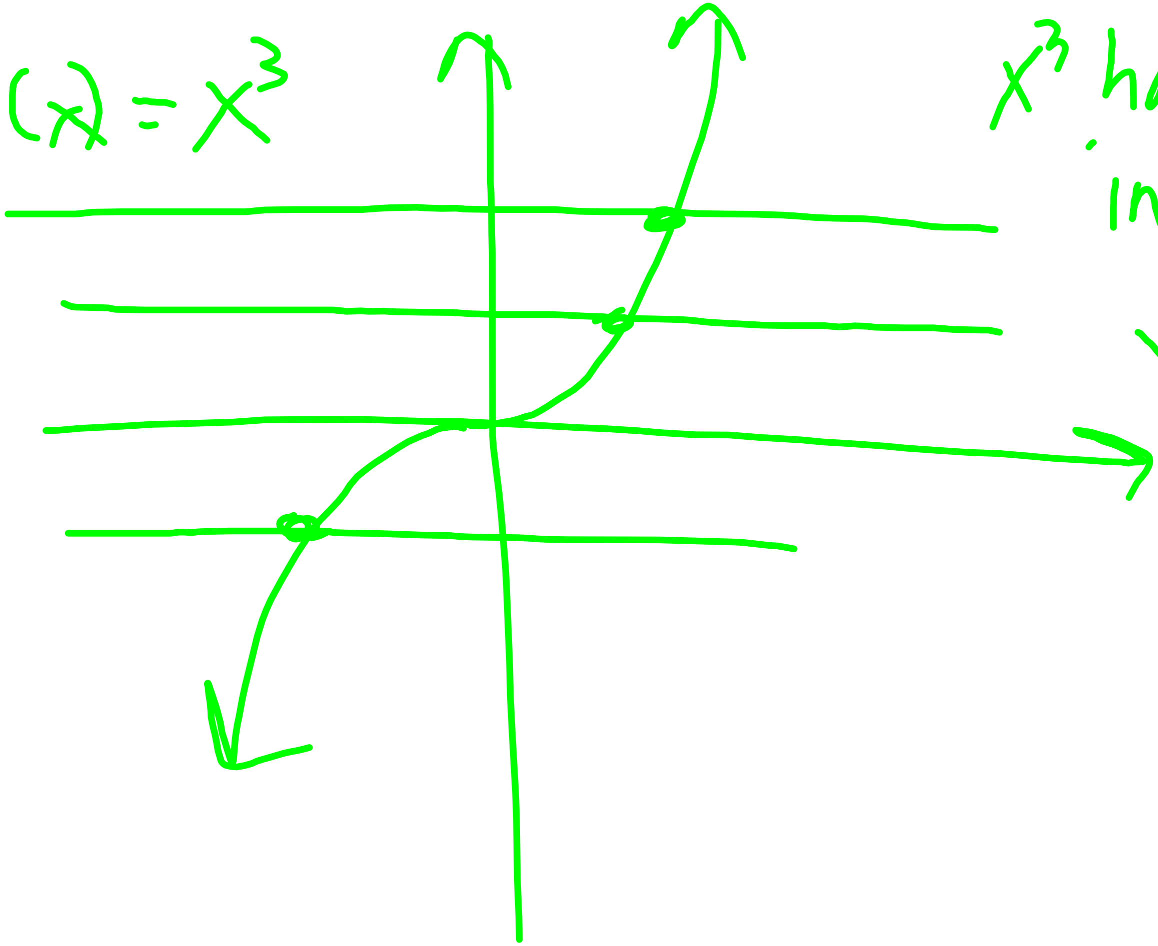
How do we tell if a function has an inverse function?

Apply the Horizontal Line Test (HLT)

If passes HLT, function has an inverse

Pass HLT

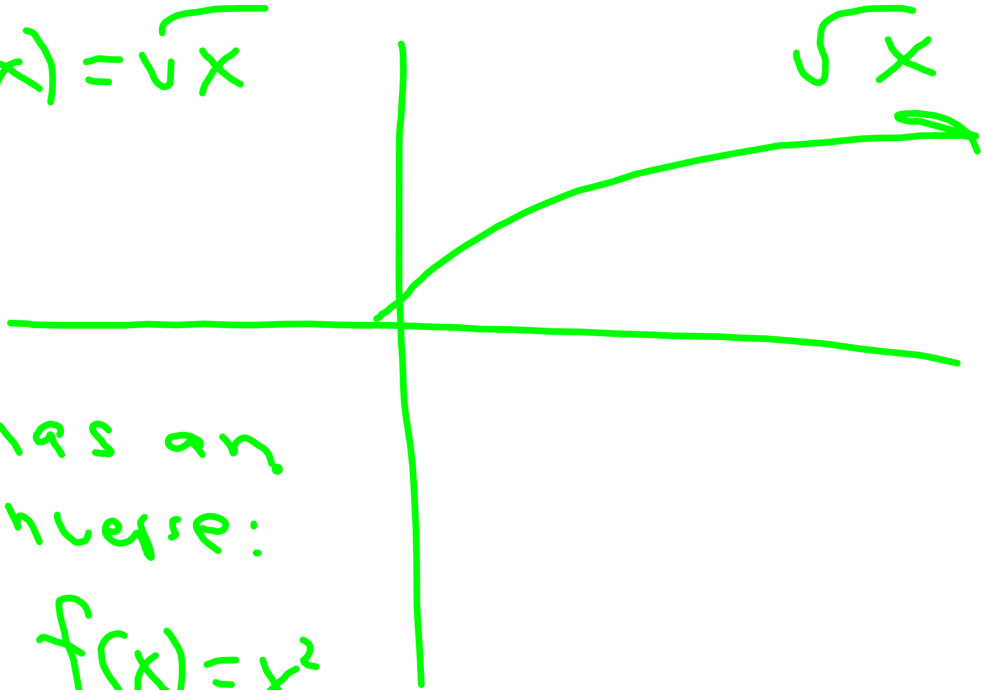
$f(x) = x^3$



means $x^3$ has an inverse: $\sqrt[3]{x}$

Pass HLT

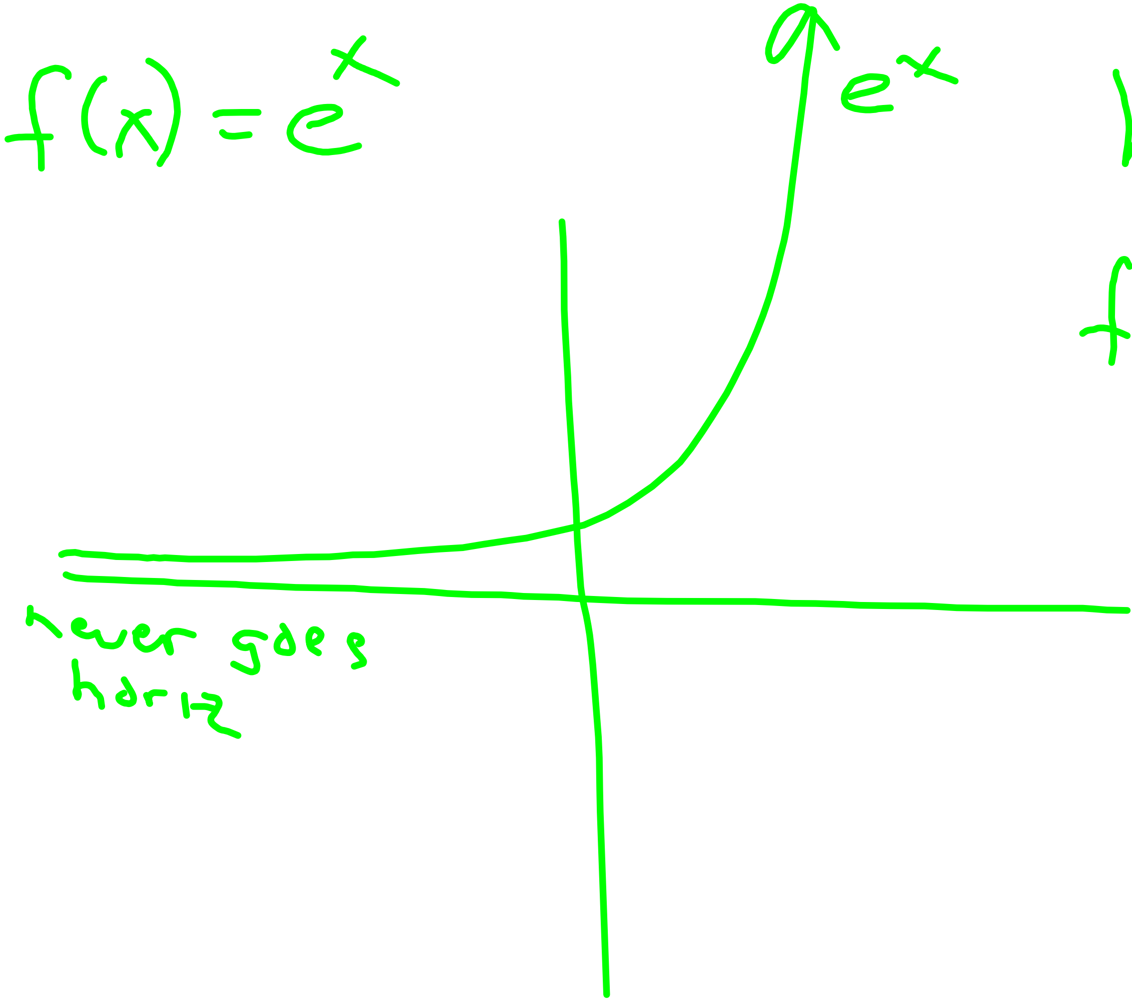
$f(x) = \sqrt{x}$




has an inverse:

$f(x) = x^2$

$f(x) = e^x$

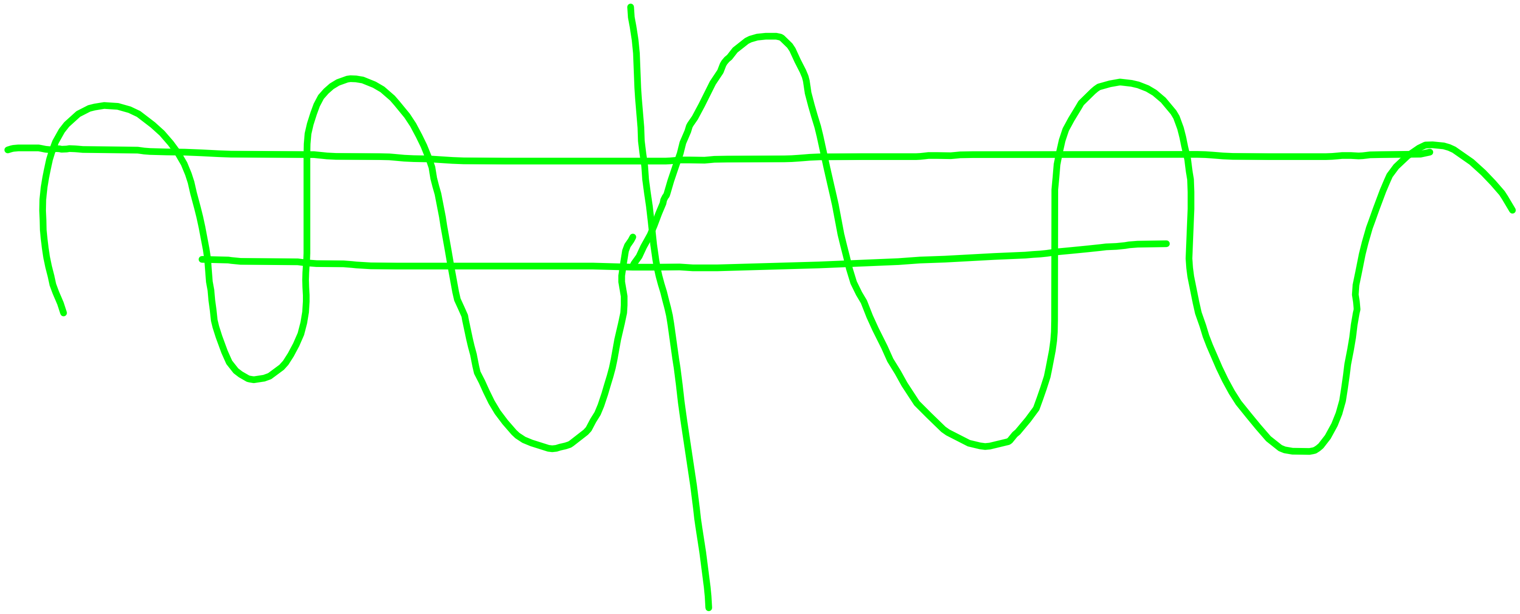


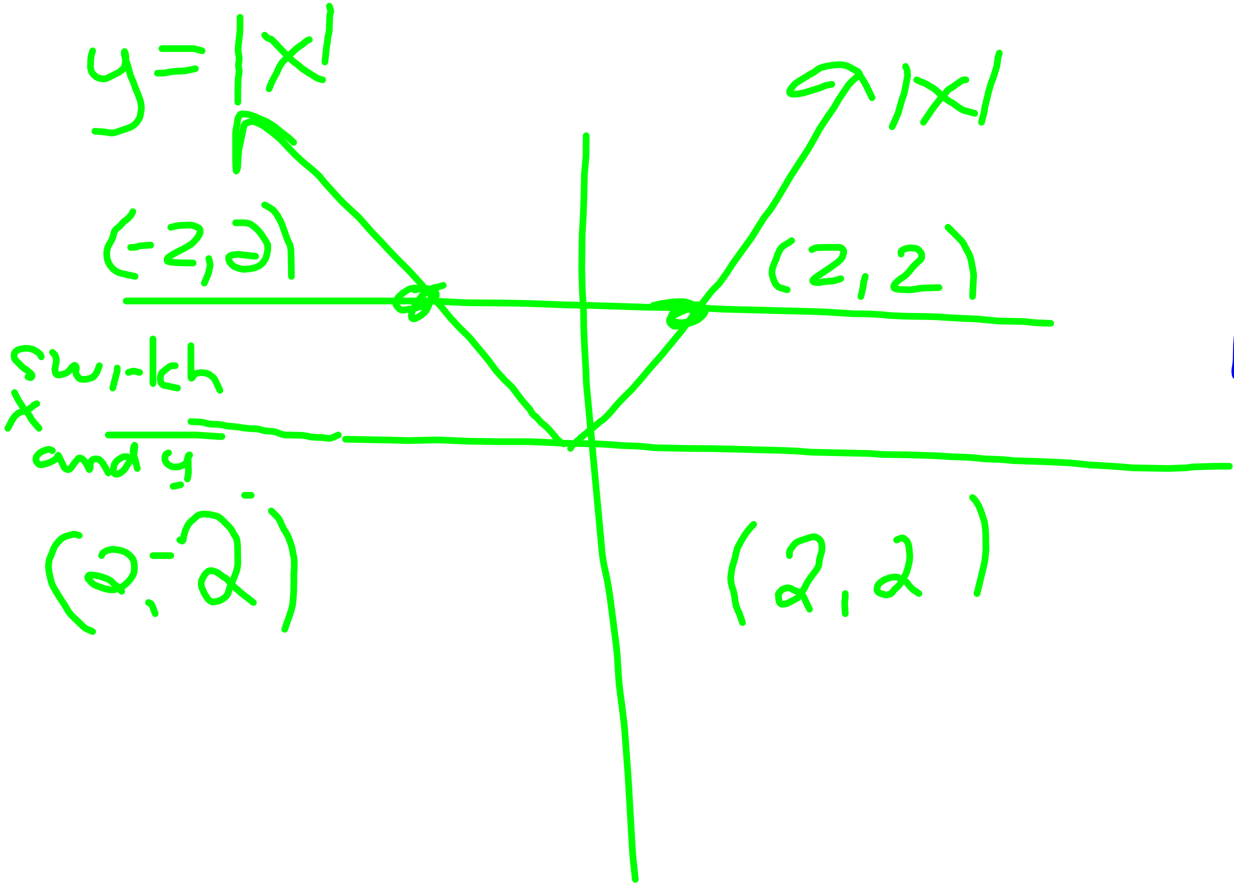

has an inverse:

$f(x) = \ln x$

fail HLT

$y = \sin x$






|x| has no inverse

$f(x) = \sqrt[3]{3x-4}$

How to get inverse fnction

$f(x) = \sqrt[3]{3x-4}$

$y = \sqrt[3]{3x-4}$ ① replace f(x) with y

$x = \sqrt[3]{3y-4}$ ② switch x and y "opposite"

$(x)^3 = (\sqrt[3]{3y-4})^3$ ③ solve for y

$x^3 = 3y - \cancel{4}$
$\quad\; +4 \quad\; \cancel{+4}$

$\frac{x^3+4}{3} = \frac{\cancel{3}y}{\cancel{3}}$

$$\frac{x^3+4}{3} = y$$

$$f^{-1}(x) = \frac{x^3+4}{3}$$

$$f(x) = \sqrt[3]{3x-4}$$

$f^{-1}$

final step

replace $y$ with

$f^{-1}(x)$

"the inverse"

$$f(x) = 2(x+5)^2$$

$$y = 2(x+5)^2$$

$$\frac{x}{2} = \frac{\cancel{2}(y+5)^2}{\cancel{2}}$$

$$\frac{x}{2} = (y+5)^2$$

$$\sqrt{\frac{x}{2}} = y+5$$

$$\sqrt{\frac{x}{2}} - 5 = y$$

$$f^{-1}(x) = \sqrt{\frac{x}{2}} - 5$$

Domain?

$$\frac{x}{2} \geq 0$$

$$\cancel{2} \cdot \frac{x}{\cancel{2}} \geq 0 \cdot 2$$

$$x \geq 0$$

$$f(x) = \frac{x-4}{3}$$

$$y = \frac{x-4}{3}$$

$$x = \frac{y-4}{3}$$

$$3x = y-4$$

$$3x+4 = y$$

$$f^{-1}(x) = 3x+4 \quad (-\infty,\infty)$$

53 > If a graph passes
55
HLT, it is called a
"one-to-one" function
which are one-to-one functions
also on 53, 55, ignore "graph the
inverse"

57 → 59

Show $f(x)$ and $g(x)$ are inverses by $f(g(x)) = x$ and $g(f(x)) = x$

58 $f(x) = \frac{x+3}{4}$ $g(x) = 4x-3$

$$f(\underbrace{g(x)}_{\text{inside}}) = f(4x-3)$$

$$= \frac{(4x-3)+3}{4}$$

$$= \frac{\cancel{4}x}{\cancel{4}}$$

$$= x$$

$$(g \circ f)(x) = g(f(x))$$

$$= g\left(\frac{x+3}{4}\right)$$

$$= \cancel{4}\left(\frac{x+3}{\cancel{4}}\right) - 3$$

$$= x + 3 - 3$$

$$= x \checkmark$$