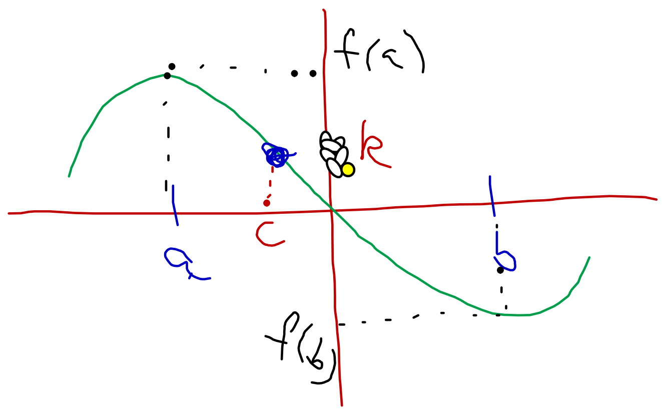
Intermediate Value Theorem IVT

If $f(x)$ is continuous on $[a,b]$ and k is a number between $f(a)$ and $f(b)$ then there is a number c in $[a,b]$ so that $f(c) = k$



$f(x)$ attains all values between $f(a)$ and $f(b)$ for some domain choice.

Ex:

x	3	7	9	10	13
$f(x)$	5	-3	2	7	-18

$f(x)$ is continuous on $[3, 13]$

$f(x) = -6$? on $[3, 13]$

yes! Due to IVT $f(x)$ has all values between 5 and -18 for some choice of x.

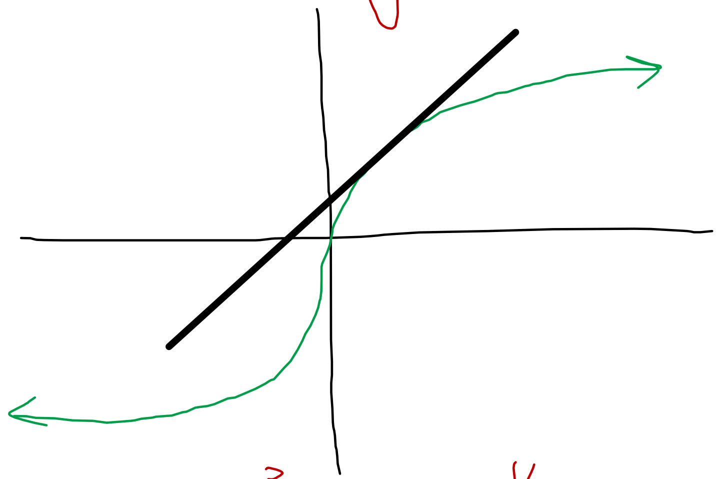
Differentiability :

Means – derivative exists

or graph of function has real value slope at each point.

Visual examples for lack of differentiability.

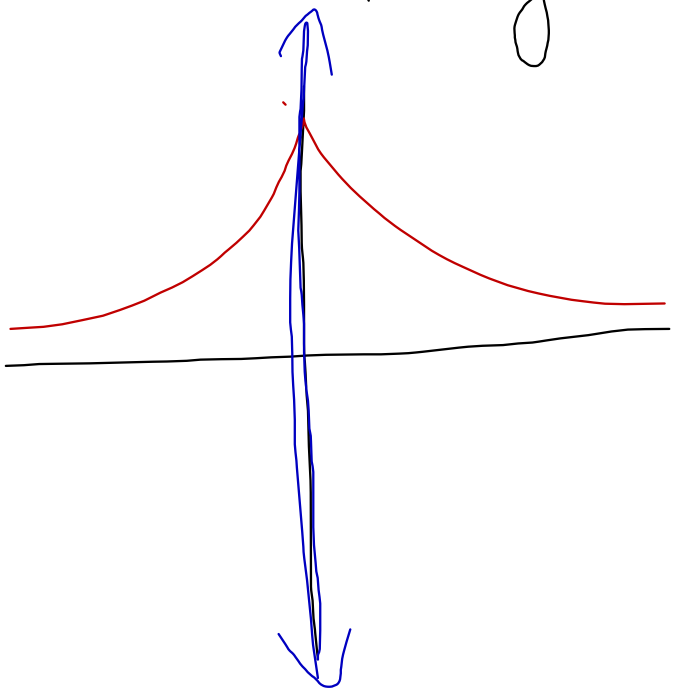
1. Vertical Tangent – No cusp.



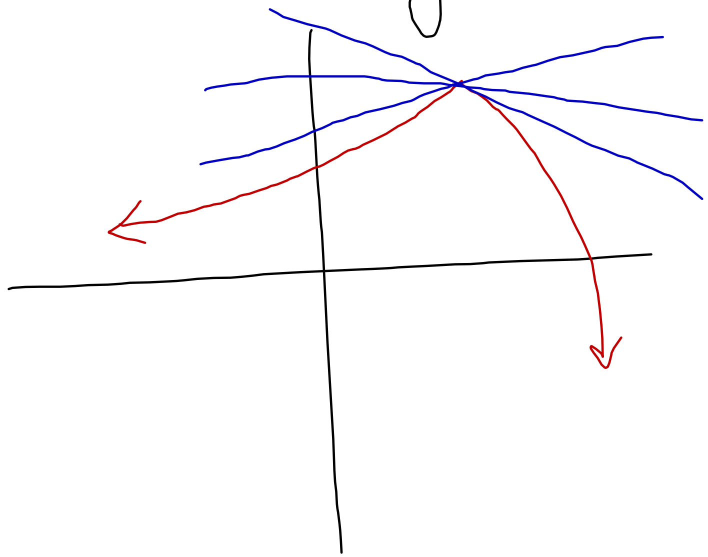
$f(x) = \sqrt[3]{x} = x^{1/3}$

$f'(x) = \frac{1}{3} x^{-2/3} = \frac{1}{3\sqrt[3]{x^2}}$

2. Vertical Tangent - Cusp.



3. No tangent - cusp



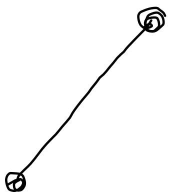
Hw pg 172 # 7-19 ~~odd~~

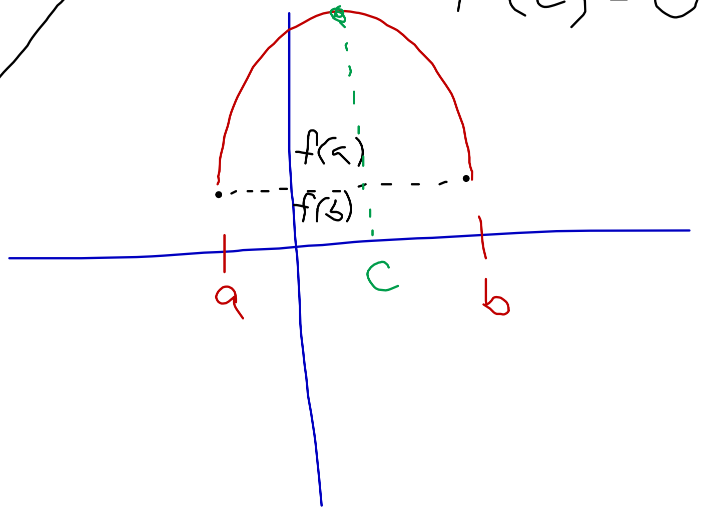
9, 13, 17, 15

Rolle's Theorem -

If $f(x)$ is continuous on $[a, b]$,
differentiable on (a, b) and $f(a) = f(b)$
then for at least one value c in $[a, b]$

$$f'(c) = 0$$





Ex: $f(x) = (x-4)^2 \quad [0, 8]$

$f(0) = (-4)^2 = 16$

$f(8) = 4^2 = 16$

Find values of c for Rolle's Thm

$f'(x) = 2(x-4) = 0$

$x = 4$ = c value!

9) $f(x) = (x-1)(x-2)(x-3) \quad [1,3]$

$f(1) = 0 \quad , \quad f(3) = 0$

$f'(x) = (x-2)(x-3) + (x-1)(x-3) + (x-1)(x-2)$

$= 3x^2 - 12x + 11 = 0$

$\dfrac{12 \pm \sqrt{12}}{6}$

$x = 2.577$

$x = 1.422$

13. $f(x) = \dfrac{x^2-2x-3}{x+2}$ $[-1, 3]$

$f(-1) = f(3) = 0$

$f'(x) = \dfrac{(x+2)(2x-2) - (x^2-2x-3)\times 1}{(x+2)^2}$

$= \dfrac{x^2+4x-1}{(x+2)^2} = 0$

$x^2+4x-1=0$

$x^2+4x+4=5$

$(x+2)^2=5$

$x+2=\pm\sqrt{5}$

$x=-2\pm\sqrt{5}$

b/c $x \in [-1,3]$

$\times$ $x=-2+\sqrt{5}$

$-2-\sqrt{5} = -2-2.2=-4.2$

$\times$ $-2+\sqrt{5} = -2+2.2 = .2$

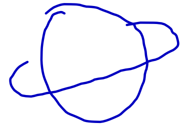
$x^2+4x-1=0$

$(x^2+4x+4)-1=0+4$

$(x+2)^2=5$

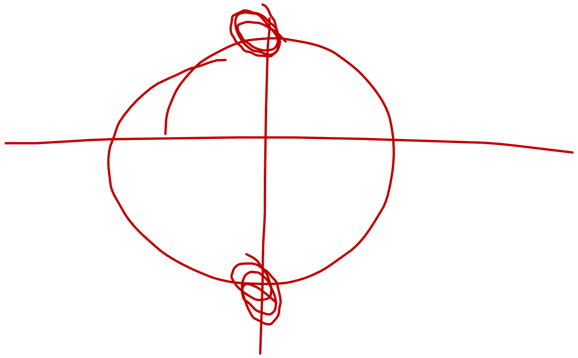
15. $f(x) = \sin x \quad [0, 2\pi]$

Rachel



$f(0) = 0, \quad f(2\pi) = 0$

$f'(x) = \cos x = 0$



$x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$

n. $f(x) = \frac{6x}{\pi} - 4\sin^2 x \qquad \left[0, \frac{\pi}{6}\right]$

$= \frac{6}{\pi} x - 4(\sin(x))^2$

$f'(x) = \frac{6}{\pi} - [\cos(x)][8\sin(x)] = 0$

$x = .2488$

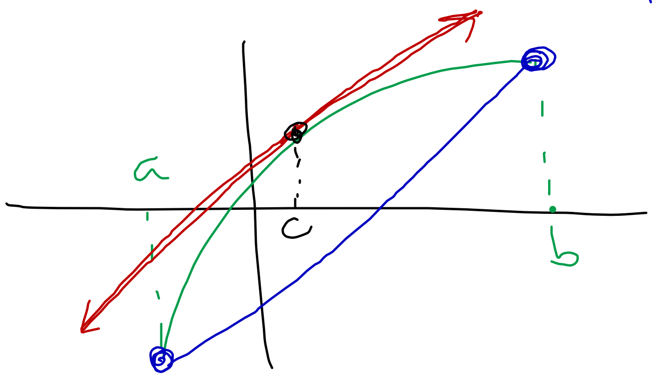
Mean Value Theorem –

If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b) then

there is a value c in $[a, b]$ so that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Tangent slope   secant slope



$f(x) = 2x^2 + x - 4 \quad [-1, 4]$

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Find c given by MVT.

#31-37

Secant

$$\frac{f(4) - f(-1)}{4+1}$$

$$\frac{32 - (-3)}{5}$$

$$= \frac{35}{5} = \boxed{7}$$

Tangent

$$\boxed{f'(x) = 4x + 1}$$

$$4x + 1 = 7$$

$$4x = 6$$

$$x = 6/4 = 3/2 = 1\tfrac{1}{2} = 1.5$$