Function Arithmetic

1. Add $(f+g)(x) = f(x)+g(x)$

Ex: $f(x) = x+4$

$g(x) = x+\frac{1}{2}$

$(f+g)(x) = (x+4)+(x+\frac{1}{2})$

$= 2x+4\frac{1}{2}$

2. Sub $(f-g)(x) = f(x)-g(x)$

3. Mult: $(fg)(x) = f(x)\cdot g(x)$

4. Div: $(f/g)(x) = \frac{f(x)}{g(x)}$

5. Composition

$$(f \circ g)(x) = f(g(x))$$

f composition g

f compose g

Ex: $f(x) = x - 2$

$g(x) = x^2 + 3x + 5$

$(f \circ g)(x) = (x^2 + 3x + 5) - 2$

$(g \circ f)(x) = (x-2)^2 + 3(x-2) + 5$

$x^2 - 4x + 4 + 3x - 6 + 5 = x^2 - x + 3$

$f(x) = x+4 \qquad g(x) = 2x+1$

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#1–7 ~~odd~~

$(g\circ f\circ g\circ g\circ f)(4) = 79$

#43–47

(No domain)

5 div

44, 43, 46

$f(4) = 8$

$g(8) = 17$

$g(17) = 35$

$f(35) = 39$

$g(39) = 79$

$$f(x) = x^2 + 6 \qquad \longrightarrow \qquad (f/g)(x) = \frac{x^2+6}{\sqrt{1-x}}$$

$$g(x) = \sqrt{1-x}$$

$$(f+g)(x) = x^2 + 6 + \sqrt{1-x}$$

$$(fg)(x) = (x^2+6)\sqrt{1-x}$$

Mult: $(x+2)(x-3) = x(x-3) + 2(x-3)$

$$= x^2 - 3x + 2x - 6 = x^2 - x - 6$$

Div: $\cancel{\dfrac{x^{\cancel{2}} + \cancel{4}}{\cancel{x} - \cancel{2}} = x+2}$

$$\frac{\cancel{x}(x-3)}{\cancel{x}(x+2)}$$

Num + Denom must be products before dividing out!

43. $f(x) = \sqrt{x+4}$ $g(x) = x^2$

9) $\sqrt{(x^2)+4}$

$(f \circ g)(x) = \sqrt{(x^2)+4}$

$\neq x+2$

10) $\left(\sqrt{x+4}\right)^2$

Do this

$(g \circ f)(x) = \left(\sqrt{x+4}\right)^2$

$= x+4$

$\sqrt{9+16} \neq 3+4 = 7$

$\sqrt{25} = 5$

$f(x) = \sqrt[3]{x-5} \qquad g(x) = x^3+1$

a. $\sqrt[3]{(x^3+1)-5} = \sqrt[3]{x^3-4}$

b. $\left(\sqrt[3]{x-5}\right)^3 + 1 = x-5+1 = x-4$

Anthony

46. a) $(\sqrt{x})^2 + 1 = x + 1$

b) $\sqrt{x^2+1} \neq x + 1$ $\qquad f(x) = x^2 + 1$

Nicole $\qquad g(x) = \sqrt{x}$

What happens when $(f \circ g)(x) = (g \circ f)(x) = x$?

It means f and g are inverses!

Ex: $f(x) = x + 2 \qquad g(x) = x - 2$

$f(3) = 3 + 2 = 5$

$g(5) = 5 - 2 = 3$

Ex $f(x) = 2x^2 - 6 \qquad g(x) = \sqrt{\dfrac{x+6}{2}}$

$f(x) = 2x^2 + 5x - 3$

$g(x) = 5x + 1$

1. $(f \circ g)(x) = 2(5x+1)^2 + 5(5x+1) - 3$

2. $(g \circ f)(x) = 5(2x^2+5x-3) + 1$

To Find an inverse formula:

Ex: $f(x) = \sqrt[3]{5x+1}$

The inverse for $f(x)$ is given by $f^{-1}(x)$.

1. Replace $f(x)$ with y

$$y = \sqrt[3]{5x+1}$$

2. Switch x and y

$$(x)^3 = \left(\sqrt[3]{5y+1}\right)^3$$

3. Solve for y.

$$x^3 = 5y+1$$

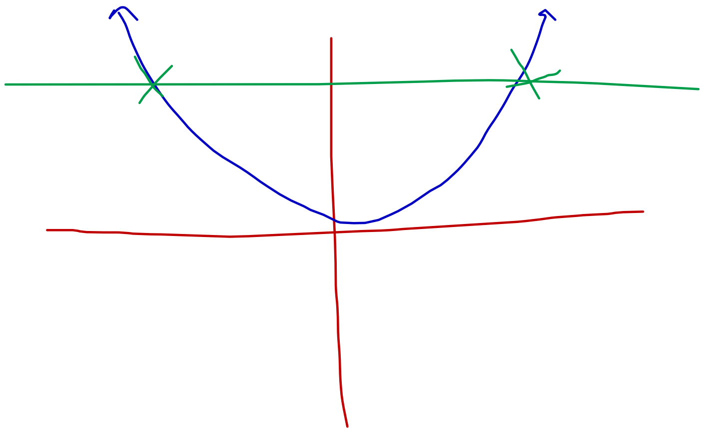
$$x^3 - 1 = 5y$$

$$\frac{x^3-1}{5} = y = f^{-1}(x)$$

Functions have inverses if they are one-to-one. (1-1)

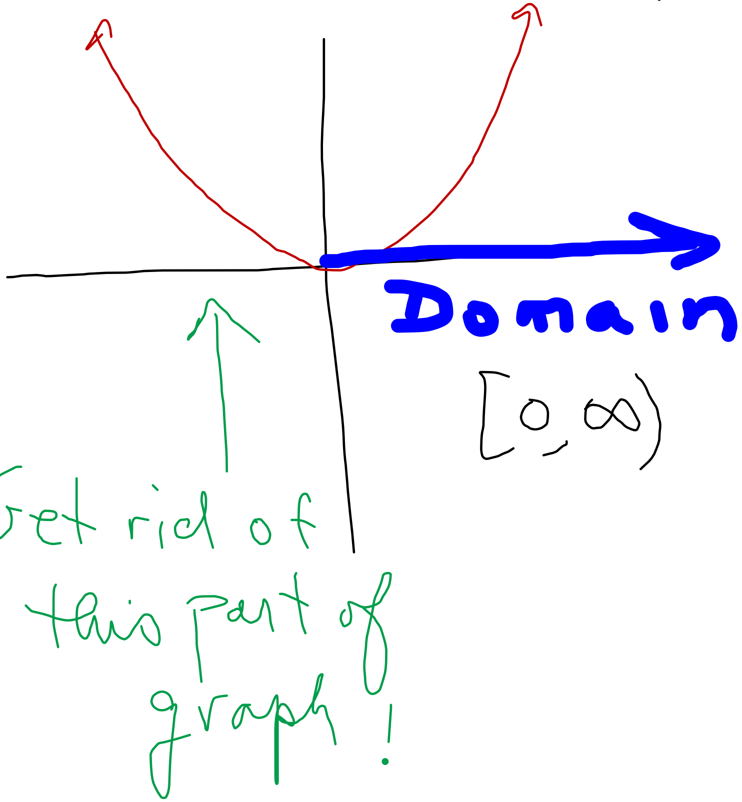
One-to-one means function graph has no points that are horizontally in line with each other.

Ex: of not 1-1



How to make a function 1-1

Ex: $f(x) = x^2, \; x \geq 0$



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35-42
Just find f^{-1}!