Ratios and Proportions

Solve Proportions If a proportion involves a variable, you can use cross products to solve the proportion. In the proportion $\frac{x}{5} = \frac{10}{13}$, x and 13 are called **extremes** and 5 and 10 are called **means**. In a proportion, the product of the extremes is equal to the product of the means.

Means-Extremes Property of Proportions	For any numbers a, b, c, and d, if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

Example **Solve $\frac{x}{5} = \frac{10}{13}$.**

$\frac{x}{5} = \frac{10}{13}$ Original proportion

$13(x) = 5(10)$ Cross products

$13x = 50$ Simplify.

$\frac{13x}{13} = \frac{50}{13}$ Divide each side by 13.

$x = 3\frac{11}{13}$ Simplify.

The solution is $3\frac{11}{13}$.

Percent of Change

Percent of Change When an increase or decrease in an amount is expressed as a percent, the percent is called the **percent of change**. If the new number is greater than the original number, the percent of change is a **percent of increase**. If the new number is less than the original number, the percent of change is the **percent of decrease**.

Example 1

Find the percent of increase.
original: 48
new: 60

First, subtract to find the amount of increase. The amount of increase is $60 - 48 = 12$.

Then find the percent of increase by using the original number, 48, as the base.

$\frac{12}{48} = \frac{r}{100}$ Percent proportion

$12(100) = 48(r)$ Cross products

$1200 = 48r$ Simplify.

$\frac{1200}{48} = \frac{48r}{48}$ Divide each side by 48.

$25 = r$ Simplify.

The percent of increase is 25%.

Example 2

Find the percent of decrease.
original: 30
new: 22

First, subtract to find the amount of decrease. The amount of decrease is $30 - 22 = 8$.

Then find the percent of decrease by using the original number, 30, as the base.

$\frac{8}{30} = \frac{r}{100}$ Percent proportion

$8(100) = 30(r)$ Cross products

$800 = 30r$ Simplify.

$\frac{800}{30} = \frac{30r}{30}$ Divide each side by 30.

$26\frac{2}{3} = r$ Simplify.

The percent of decrease is $26\frac{2}{3}\%$, or about 27%.

Percent of Change

Solve Problems Discounted prices and prices including tax are applications of percent of change. Discount is the amount by which the regular price of an item is reduced. Thus, the discounted price is an example of percent of decrease. Sales tax is amount that is added to the cost of an item, so the price including tax is an example of percent of increase.

Example **A coat is on sale for 25% off the original price. If the original price of the coat is $75, what is the discounted price?**

The discount is 25% of the original price.

25% of $75 = 0.25 × 75   25% = 0.25

= 18.75   Use a calculator.

Subtract $18.75 from the original price.

$75 − $18.75 = $56.25

The discounted price of the coat is $56.25.

Solving Equations and Formulas

Solve for Variables Sometimes you may want to solve an equation such as $V = \ell wh$ for one of its variables. For example, if you know the values of V, w, and h, then the equation $\ell = \frac{V}{wh}$ is more useful for finding the value of ℓ. If an equation that contains more than one variable is to be solved for a specific variable, use the properties of equality to isolate the specified variable on one side of the equation.

Example 1 **Solve $2x - 4y = 8$ for y.**

$$\begin{aligned} 2x - 4y &= 8 \\ 2x - 4y - 2x &= 8 - 2x \\ -4y &= 8 - 2x \\ \frac{-4y}{-4} &= \frac{8 - 2x}{-4} \\ y &= \frac{8 - 2x}{-4} \text{ or } \frac{2x - 8}{4} \end{aligned}$$

The value of y is $\frac{2x - 8}{4}$.

Example 2 **Solve $3m - n = km - 8$ for m.**

$$\begin{aligned} 3m - n &= km - 8 \\ 3m - n - km &= km - 8 - km \\ 3m - n - km &= -8 \\ 3m - n - km + n &= -8 + n \\ 3m - km &= -8 + n \\ m(3 - k) &= -8 + n \\ \frac{m(3 - k)}{3 - k} &= \frac{-8 + n}{3 - k} \\ m &= \frac{-8 + n}{3 - k}, \text{ or } \frac{n - 8}{3 - k} \end{aligned}$$

The value of m is $\frac{n - 8}{3 - k}$. Since division by 0 is undefined, $3 - k \neq 0$, or $k \neq 3$.

Solving Equations and Formulas

Use Formulas Many real-world problems require the use of formulas. Sometimes solving a formula for a specified variable will help solve the problem.

Example **The formula $C = \pi d$ represents the circumference of a circle, or the distance around the circle, where d is the diameter. If an airplane could fly around Earth at the equator without stopping, it would have traveled about 24,900 miles. Find the diameter of Earth.**

$C = \pi d$ Given formula

$d = \frac{C}{\pi}$ Solve for d.

$d = \frac{24{,}900}{3.14}$ Use $\pi = 3.14$.

$d \approx 7930$ Simplify.

The diameter of Earth is about 7930 miles.

Weighted Averages

Mixture Problems

Weighted Average	The weighted average M of a set of data is the sum of the product of each number in the set and its weight divided by the sum of all the weights.

Mixture Problems are problems where two or more parts are combined into a whole. They involve weighted averages. In a mixture problem, the weight is usually a price or a percent of something.

Example **Delectable Cookie Company sells chocolate chip cookies for $6.95 per pound and white chocolate cookies for $5.95 per pound. How many pounds of chocolate chip cookies should be mixed with 4 pounds of white chocolate cookies to obtain a mixture that sells for $6.75 per pound.**

Let w = the number of pounds of chocolate chip cookies

	Number of Pounds	Price per Pound	Total Price
Chocolate Chip	w	6.95	$6.95w$
White Chocolate	4	5.95	4(5.95)
Mixture	$w + 4$	6.75	$6.75(w + 4)$

Equation: $6.95w + 4(5.95) = 6.75(w + 4)$

Solve the equation.

$6.95w + 4(5.95) = 6.75(w + 4)$ Original equation

$6.95w + 23.80 = 6.75w + 27$ Simplify.

$6.95w + 23.80 - 6.75w = 6.75w + 27 - 6.75w$ Subtract $6.75w$ from each side.

$0.2w + 23.80 = 27$ Simplify.

$0.2w + 23.80 - 23.80 = 27 - 23.80$ Subtract 23.80 from each side.

$0.2w = 3.2$ Simplify.

$w = 16$ Simplify.

16 pounds of chocolate chip cookies should be mixed with 4 pounds of white chocolate cookies.

Weighted Averages

Uniform Motion Problems Motion problems are another application of weighted averages. **Uniform motion problems** are problems where an object moves at a certain speed, or rate. Use the formula $d = rt$ to solve these problems, where d is the distance, r is the rate, and t is the time.

Example **Bill Gutierrez drove at a speed of 65 miles per hour on an expressway for 2 hours. He then drove for 1.5 hours at a speed of 45 miles per hour on a state highway. What was his average speed?**

$M = \frac{65 \cdot 2 + 45 \cdot 1.5}{2 + 1.5}$ Definition of weighted average

≈ 56.4 Simplify.

Bill drove at an average speed of about 56.4 miles per hour.