

5-1 Study Guide and Intervention

Slope

Find Slope

Slope of a Line	$m = \frac{\text{rise}}{\text{run}}$ or $m = \frac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1) and (x_2, y_2) are the coordinates of any two points on a nonvertical line
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Example 1 Find the slope of the line that passes through $(-3, 5)$ and $(4, -2)$.

Let $(-3, 5) = (x_1, y_1)$ and $(4, -2) = (x_2, y_2)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope formula} \\ &= \frac{-2 - 5}{4 - (-3)} && y_2 = -2, y_1 = 5, x_2 = 4, x_1 = -3 \\ &= \frac{-7}{7} && \text{Simplify.} \\ &= -1 \end{aligned}$$

Example 2 Find the value of r so that the line through $(10, r)$ and $(3, 4)$ has a slope of $-\frac{2}{7}$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Slope formula} \\ -\frac{2}{7} &= \frac{4 - r}{3 - 10} && m = -\frac{2}{7}, y_2 = 4, y_1 = r, x_2 = 3, x_1 = 10 \\ -\frac{2}{7} &= \frac{4 - r}{-7} && \text{Simplify.} \\ -2(-7) &= 7(4 - r) && \text{Cross multiply.} \\ 14 &= 28 - 7r && \text{Distributive Property} \\ -14 &= -7r && \text{Subtract 28 from each side.} \\ 2 &= r && \text{Divide each side by } -7. \end{aligned}$$

Exercises

Find the slope of the line that passes through each pair of points.

- | | | |
|----------------------|-------------------------|--------------------------|
| 1. $(4, 9), (1, 6)$ | 2. $(-4, -1), (-2, -5)$ | 3. $(-4, -1), (-4, -5)$ |
| 4. $(2, 1), (8, 9)$ | 5. $(14, -8), (7, -6)$ | 6. $(4, -3), (8, -3)$ |
| 7. $(1, -2), (6, 2)$ | 8. $(2, 5), (6, 2)$ | 9. $(4, 3.5), (-4, 3.5)$ |

Determine the value of r so the line that passes through each pair of points has the given slope.

- | | | |
|---|---|--|
| 10. $(6, 8), (r, -2), m = 1$ | 11. $(-1, -3), (7, r), m = \frac{3}{4}$ | 12. $(2, 8), (r, -4), m = -3$ |
| 13. $(7, -5), (6, r), m = 0$ | 14. $(r, 4), (7, 1), m = \frac{3}{4}$ | 15. $(7, 5), (r, 9), m = 6$ |
| 16. $(10, r), (3, 4), m = -\frac{2}{7}$ | 17. $(10, 4), (-2, r), m = -0.5$ | 18. $(r, 3), (7, r), m = -\frac{1}{5}$ |

Slope-Intercept Form

Example 2

Graph $3x - 4y = 8$.

$$3x - 4y = 8$$

Original equation

$$-4y = -3x + 8$$

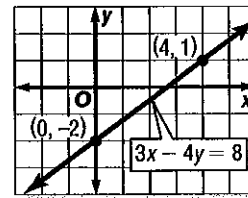
Subtract $3x$ from each side.

$$\frac{-4y}{-4} = \frac{-3x + 8}{-4}$$

Divide each side by -4 .

$$y = \frac{3}{4}x - 2$$

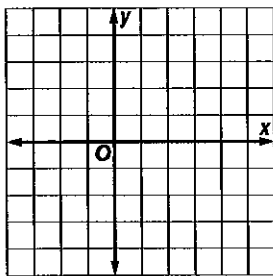
Simplify.



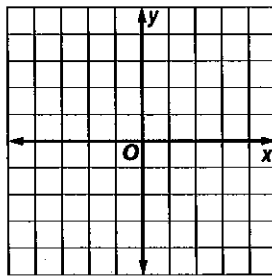
The y -intercept of $y = \frac{3}{4}x - 2$ is -2 and the slope is $\frac{3}{4}$. So graph the point $(0, -2)$. From this point, move up 3 units and right 4 units. Draw a line passing through both points.

Graph each equation.

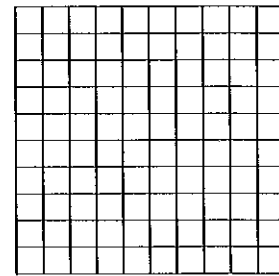
8. $y = -\frac{1}{2}x + 2$



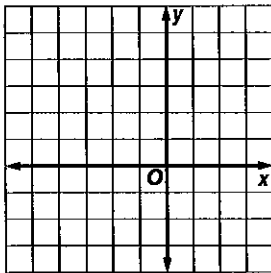
9. $3y = 2x - 6$



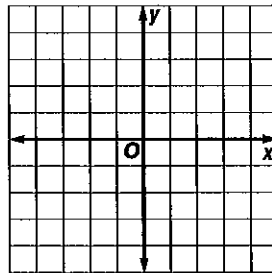
10. $6x + 3y = 6$



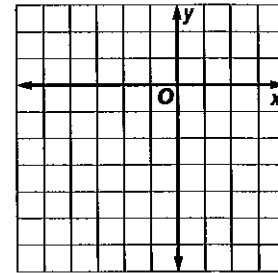
12. $y = x + 4$



13. $y = -2x - 1$



14. $x + y = -3$



Write a linear equation in slope-intercept form to model each situation.

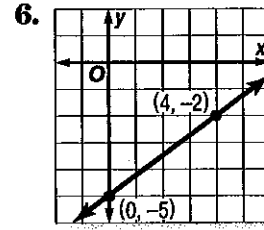
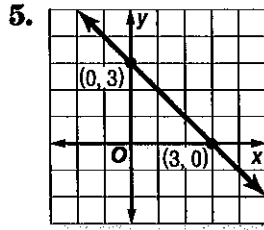
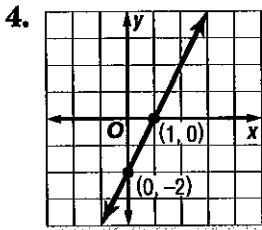
15. A video store charges \$10 for a rental card plus \$2 per rental.

16. A Norfolk pine is 18 inches tall and grows at a rate of 1.5 feet per year.

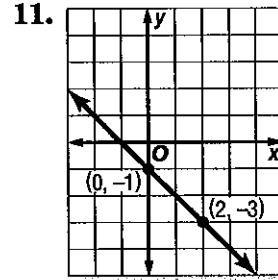
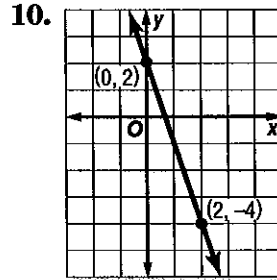
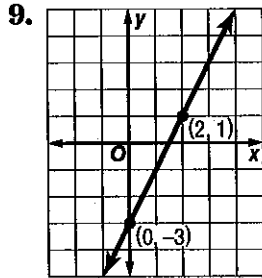
17. A Cairn terrier weighs 30 pounds and is on a special diet to lose 2 pounds per month.

18. An airplane at an altitude of 3000 feet descends at a rate of 500 feet per mile.

Write an equation of the line shown in each graph.



Write an equation of the line shown in each graph.



Example 1 Write an equation of a line that passes through $(-4, 2)$ with slope 3.

The line has slope 3. To find the y -intercept, replace m with 3 and (x, y) with $(-4, 2)$ in the slope-intercept form. Then solve for b .

$$\begin{aligned} y &= mx + b && \text{Slope-intercept form} \\ 2 &= 3(-4) + b && m = 3, y = 2, \text{ and } x = -4 \\ 2 &= -12 + b && \text{Multiply.} \\ 14 &= b && \text{Add 12 to each side.} \end{aligned}$$

Therefore, the equation is $y = 3x + 14$.

Example 2 Write an equation of the line that passes through $(-2, -1)$ with slope $\frac{1}{4}$.

The line has slope $\frac{1}{4}$. Replace m with $\frac{1}{4}$ and (x, y) with $(-2, -1)$ in the slope-intercept form.

$$\begin{aligned} y &= mx + b && \text{Slope-intercept form} \\ -1 &= \frac{1}{4}(-2) + b && m = \frac{1}{4}, y = -1, \text{ and } x = -2 \\ -1 &= -\frac{1}{2} + b && \text{Multiply.} \\ -\frac{1}{2} &= b && \text{Add } \frac{1}{2} \text{ to each side.} \end{aligned}$$

Therefore, the equation is $y = \frac{1}{4}x - \frac{1}{2}$.

4. $(1, 9), m = 4$

5. $(4, 2), m = -2$

6. $(2, -2), m = 3$

7. $(3, 0), m = 5$

8. $(-3, -2), m = 2$

9. $(-5, 4), m = -4$

Example

Write an equation of the line that passes through (1, 2) and (3, -2).

Find the slope m . To find the y -intercept, replace m with its computed value and (x, y) with (1, 2) in the slope-intercept form. Then solve for b .

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{Slope formula}$$

$$m = \frac{-2 - 2}{3 - 1} \quad y_2 = -2, y_1 = 2, x_2 = 3, x_1 = 1$$

$$m = -2 \quad \text{Simplify.}$$

$$y = mx + b \quad \text{Slope-intercept form}$$

$$2 = -2(1) + b \quad \text{Replace } m \text{ with } -2, y \text{ with } 2, \text{ and } x \text{ with } 1.$$

$$2 = -2 + b \quad \text{Multiply.}$$

$$4 = b \quad \text{Add 2 to each side.}$$

Therefore, the equation is $y = -2x + 4$.

4. (-1, 6), (7, -10)

5. (0, 2), (1, 7)

6. (6, -25), (-1, 3)

7. (-2, -1), (2, 11)

8. (10, -1), (4, 2)

9. (-14, -2), (7, 7)

Example 1

Write the point-slope form of an equation for a line that passes through (6, 1) and has a slope of $-\frac{5}{2}$.

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$

$$y - 1 = -\frac{5}{2}(x - 6) \quad m = -\frac{5}{2}; (x_1, y_1) = (6, 1)$$

Therefore, the equation is $y - 1 = -\frac{5}{2}(x - 6)$.

4. (2, 1), $m = 4$

5. (-7, 2), $m = 6$

6. (8, 3), $m = 1$

7. (-6, 7), $m = 0$

8. (4, 9), $m = \frac{3}{4}$

9. (-4, -5), $m = -\frac{1}{2}$

Example 2

Write the point-slope form of an equation for a horizontal line that passes through (4, -1).

$$y - y_1 = m(x - x_1) \quad \text{Point-slope form}$$

$$y - (-1) = 0(x - 4) \quad m = 0; (x_1, y_1) = (4, -1)$$

$$y + 1 = 0 \quad \text{Simplify.}$$

Therefore, the equation is $y + 1 = 0$.

Parallel Lines Two nonvertical lines are **parallel** if they have the same slope. All vertical lines are parallel.

Write the slope-intercept form for an equation of the line that passes through the given point and is parallel to the graph of each equation.

4. (-2, 2), $y = 4x - 2$

5. (6, 4), $y = \frac{1}{3}x + 1$

6. (4, -2), $y = -2x + 3$

7. (-2, 4), $y = -3x + 10$

8. (-1, 6), $3x + y = 12$

9. (4, -6), $x + 2y = 5$

Perpendicular Lines Two lines are **perpendicular** if their slopes are negative reciprocals of each other. Vertical and horizontal lines are perpendicular.

Example

Write the slope-intercept form for an equation that passes through $(-4, 2)$ and is perpendicular to the graph of $2x - 3y = 9$.

Find the slope of $2x - 3y = 9$.

$$\begin{aligned}
 2x - 3y &= 9 && \text{Original equation} \\
 -3y &= -2x + 9 && \text{Subtract } 2x \text{ from each side.} \\
 y &= \frac{2}{3}x - 3 && \text{Divide each side by } -3.
 \end{aligned}$$

The slope of $y = \frac{2}{3}x - 3$ is $\frac{2}{3}$. So, the slope of the line passing through $(-4, 2)$ that is perpendicular to this line is the negative reciprocal of $\frac{2}{3}$, or $-\frac{3}{2}$.
Use the point-slope form to find the equation.

Write the slope-intercept form for an equation of the line that passes through the given point and is perpendicular to the graph of each equation.

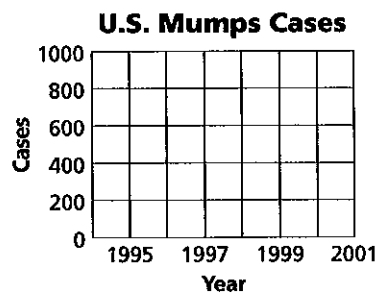
1. $(4, 2), y = \frac{1}{2}x + 1$
2. $(2, -3), y = -\frac{2}{3}x + 4$
3. $(6, 4), y = 7x + 1$
4. $(-8, -7), y = -x - 8$
5. $(6, -2), y = -3x - 6$
6. $(-5, -1), y = \frac{5}{2}x - 3$
7. $(-9, -5), y = -3x - 1$
8. $(-1, 3), 2x + 4y = 12$
9. $(6, -6), 3x - y = 6$

DISEASE For Exercises 3–6, use the table that shows the number of cases of mumps in the United States for the years 1995 to 1999.

U.S. Mumps Cases					
Year	1995	1996	1997	1998	1999
Cases	906	751	683	666	387

Source: Centers for Disease Control and Prevention

3. Draw a scatter plot and determine what relationship, if any, exists in the data.
4. Draw a line of fit for the scatter plot.
5. Write the slope-intercept form of an equation for the line of fit.
6. Predict the number of cases in 2004.

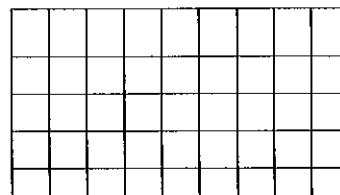


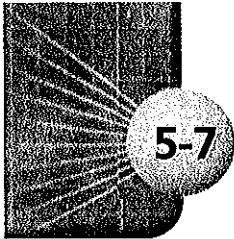
ZOOS For Exercises 7–10, use the table that shows the average and maximum longevity of various animals in captivity.

Longevity (years)								
Avg.	12	25	15	8	35	40	41	20
Max.	47	50	40	20	70	77	61	54

Source: Walker's Mammals of the World

7. Draw a scatter plot and determine what relationship, if any, exists in the data.
8. Draw a line of fit for the scatter plot.
9. Write the slope-intercept form of an equation for the





5-7 Study Guide and Intervention *(continued)*

Scatter Plots and Lines of Fit

Lines of Fit

Example

The table below shows the number of students per computer in United States public schools for certain school years from 1990 to 2000.

Year	1990	1992	1994	1996	1998	2000
Students per Computer	22	18	14	10	6.1	5.4

- a. Draw a scatter plot and determine what relationship exists, if any.

Since y decreases as x increases, the correlation is negative.

- b. Draw a line of fit for the scatter plot.

Draw a line that passes close to most of the points. A line of fit is shown.

- c. Write the slope-intercept form of an equation for the line of fit.

The line of fit shown passes through (1993, 16) and (1999, 5.7). Find the slope.

$$m = \frac{5.7 - 16}{1999 - 1993}$$

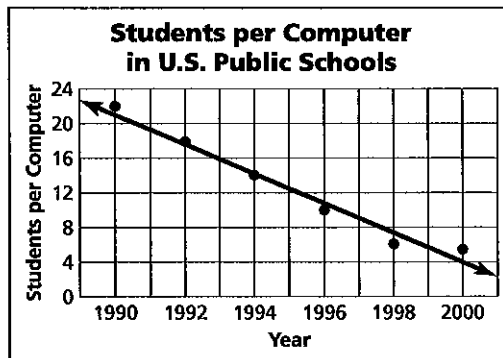
$$m = -1.7$$

Find b in $y = -1.7x + b$.

$$16 = -1.7 \cdot 1993 + b$$

$$3404 = b$$

Therefore, an equation of a line of fit is $y = -1.7x + 3404$.



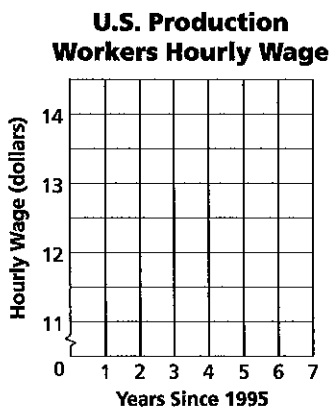
Source: *The World Almanac*

Exercises

Refer to the table for Exercises 1–3.

Years Since 1995	Hourly Wage
0	\$11.43
1	\$11.82
2	\$12.28
3	\$12.78
4	\$13.24

1. Draw a scatter plot.
2. Draw a line of fit for the data.
3. Write the slope-intercept form of an equation for the line of fit.



Source: *The World Almanac*