

Savings and Loans Review

① a) let A_n = acct balance (\$) after n quarters

$$\begin{cases} A_0 = 400 \\ A_n = \left(1 + \frac{.012}{4}\right) A_{n-1} + 75 \end{cases}$$

b) $A_{12} = 1329.64$

The account balance is \$1,329.64 after 3 years.

c) Interest Earned = Balance - [(Quarterly Pmt) (# of Quarters) + Principal]

$$\begin{aligned} &= 1329.64 - [75(12) + 400] \\ &= 29.64 \end{aligned}$$

After 3 years, I have earned \$29.64 in interest.

d) $A_{20} = 1968.22$

$A_{21} = 2049.12$

The account is worth at least \$2000 after 63 months, or 5 years and 3 months.

★ Saying 21 quarters is not "user friendly."

2) let B_n = loan balance (\$) after n months

$$\begin{cases} B_0 = 15700 \\ B_n = (1 + \frac{.061}{12})B_{n-1} - \text{pmt} \end{cases}$$

If $\text{pmt} = 175.09$, then $B_{120} = 0.26$

If $\text{pmt} = 175.10$, then $B_{120} = -1.36$

A monthly payment of \$175.10 is needed in order to repay the loan in 10 years.

$$\begin{aligned} \text{b) Total Paid} &= (\text{Monthly Pmt})(\# \text{ of Months}) - \text{overpmt} \\ &= 175.10(120) - 1.36 \\ &= 21010.64 \end{aligned}$$

You actually pay a total of \$21,010.64 for the PV system.

③ a) Let $H_n =$ mortgage balance (\$) after n months

$$\begin{cases} H_0 = \text{principal} \\ H_n = \left(1 + \frac{0.06375}{12}\right) H_{n-1} - 1700 \end{cases}$$

If principal = 272,493, $H_{360} = 2.04$

If principal = 272,492, $H_{360} = -4.61$

With a monthly payment of \$1,700, I can afford a mortgage of \$272,492.

b) Interest Paid = Total Paid - Principal

$$= \left[\left(\frac{\text{Monthly Pmt}}{\text{Months}} \right) (\# \text{ of Months}) - \text{overpmt} \right] - \text{Principal}$$

$$= 1700(360) - 4.61 - 272492$$

$$= 339503.39$$

I pay \$339,503.39 in interest on the loan.

★ If you went to the nearest penny in part (a), your numbers will of course be different \Rightarrow \$272,492.71
\$339,507.29

③ c) let $C_n =$ mortgage balance (\$) after n months

$$\begin{cases} C_0 = 272492 \\ C_n = (1 + \frac{.06375}{12}) C_{n-1} - 1750 \end{cases}$$

$$C_{331} = 619.37$$

$$C_{332} = -1127.34$$

$$\text{Diff in Loan Length} = \text{Orig Loan Length} - \text{New Loan Length} = 360 - 332 = 28$$

$$\begin{aligned} \text{Orig Total Paid} &= (\text{Monthly Pmt}) (\# \text{ of Months}) - \text{overpmt} \\ &= 1700(360) - 4.61 = 611995.39 \end{aligned}$$

$$\begin{aligned} \text{New Total Paid} &= (\text{Monthly Pmt}) (\# \text{ of Months}) - \text{overpmt} \\ &= 1750(332) - 1127.34 = 579872.66 \end{aligned}$$

$$\begin{aligned} \text{Savings} &= \text{Orig Total Paid} - \text{New Total Paid} \\ &= 611995.39 - 579872.66 = 32122.73 \end{aligned}$$

By paying an additional \$50 per month, I pay the loan off 28 months quicker and save \$32,122.73.

★ If nearest penny in part (a) \Rightarrow 28 months \$32,123.06