

① given $f(x) = \sqrt{4-x^2}$ and $g(x) = \frac{x^2}{x+1}$,
find $(\frac{f}{g})(x)$ and $\text{Dom} \frac{f}{g}$

② given $f(x) = x^4$ and $g(x) = \frac{x}{x+2}$,
find $(g-f)(x)$, $(\frac{f}{g})(x)$, and $\text{Dom} \frac{f}{g}$

③ given $f(x) = \sqrt{x^2-9}$ on $(-\infty, -3]$,
find f^{-1} and graph f and f^{-1}

$$\textcircled{10} \quad f(x) = \sqrt{x^2 - 4} \quad g(x) = \frac{x^2}{x^2 + 1}$$

$$a) \quad (f+g)(x) = \sqrt{x^2 - 4} + \frac{x^2}{x^2 + 1}$$

$$b) \quad (f-g)(x) = \sqrt{x^2 - 4} - \frac{x^2}{x^2 + 1}$$

$$c) \quad (fg)(x) = (\sqrt{x^2 - 4}) \left(\frac{x^2}{x^2 + 1} \right)$$

$$= \frac{x^2 \sqrt{x^2 - 4}}{x^2 + 1}$$

$$d) \quad \left(\frac{f}{g} \right)(x) = \frac{\frac{\sqrt{x^2 - 4}}{x^2}}{x^2 + 1}$$

$$= \frac{\frac{\sqrt{x^2 - 4}}{x^2}}{x^2 + 1} \cdot \frac{x^2 + 1}{x^2 + 1}$$

$$= \frac{(x^2 + 1) \sqrt{x^2 - 4}}{x^2}$$

$$\text{Dom } f = (-\infty, -2] \cup [2, \infty)$$

$$\text{Dom } g = \mathbb{R}$$

$$g(x) = 0 \text{ for } x = 0$$

$$\text{so, } \text{Dom } \frac{f}{g} = (-\infty, -2] \cup [2, \infty) - \{0\}$$

$$= (-\infty, -2] \cup [2, \infty)$$

$$\textcircled{12} \quad f(x) = \frac{x}{x+1} \quad \text{and} \quad g(x) = x^3$$

$$\begin{aligned} \text{a) } (f+g)(x) &= \frac{x}{x+1} + x^3 = \frac{x}{x+1} + \frac{x^3(x+1)}{x+1} \\ &= \frac{x^4 + x^3 + x}{x+1} \end{aligned}$$

$$\begin{aligned} \text{b) } (f-g)(x) &= \frac{x}{x+1} - x^3 = \frac{x}{x+1} - \frac{x^3(x+1)}{x+1} \\ &= \frac{-x^4 - x^3 + x}{x+1} \end{aligned}$$

$$\text{c) } (fg)(x) = \frac{x}{x+1} (x^3) = \frac{x^4}{x+1}$$

$$\begin{aligned} \text{d) } \left(\frac{f}{g}\right)(x) &= \frac{\frac{x}{x+1}}{x^3} = \frac{x}{x+1} \cdot \frac{1}{x^3} \\ &= \frac{1}{x+1} \cdot \frac{1}{x^2} \\ &= \frac{1}{x^3 + x^2} \end{aligned}$$

$$\text{Dom } f = \mathbb{R} - \{-1\}$$

$$\text{Dom } g = \mathbb{R}$$

$$g(x) = 0 \quad \text{for} \quad x = 0$$

$$\text{so, } \text{Dom } \frac{f}{g} = \mathbb{R} - \{-1, 0\}$$