Recursion WS 1 Answers

1. a) geometric common ratio = 2
   b) none c) Fibonacci
   d) arithmetic common difference = 3
   e) none f) Fibonacci
   g) arithmetic common difference = –7
   h) geometric common ratio = –2
   i) none j) arithmetic common difference = 0
   (You could also say this sequence is geometric with a common ratio of 1, but most mathematicians would not count this as geometric. In fact, most would not count this as an arithmetic sequence, either. They would say "The terms of the sequence are constant.")

2. a) These sequences are arithmetic since the difference between consecutive terms is constant.
   b) The first sequence consists of the odd natural numbers.
   c) The second sequence consists of the even natural numbers.

3. a) The height of the candle decreases by 2 centimeters per hour.
   b) The candle's original height was 21 cm.
   c) The number 0 is not one of the terms in the sequence.
   d) The candle will last $10\frac{1}{2}$ hours.
   e) The answers for parts (c) and (d) do not contradict each other since the sequence is simply a discrete model for the candle's height, which is actually a continuous function of time.

4. a) The first term is 1. Each succeeding term is 3 times the preceding term.
   b) The first term is 1. Each succeeding term is 4 times the preceding term.
   c) These are geometric sequences.

5. a) 3 8 13 18 $\underline{23}$ $\underline{28}$
   c) 13 17 $\underline{21}$ 25 $\underline{29}$ $\underline{33}$
   e) 4 $\underline{16}$ 28 $\underline{40}$ 52

6. a) The common ratio for this sequence is $r = 2$.
   b) $\underline{3}$ $\underline{6}$ $\underline{12}$ 24 48 $\underline{96}$ $\underline{192}$ $\underline{384}$
   c) Crossing out every other term still produces a geometric sequence in the remaining terms.

7. a) 3 12 48 $\underline{192}$ $\underline{768}$
   b) 256 192 $\underline{144}$ 108 $\underline{81}$ $\underline{\frac{243}{4}}$
   c) 16 $\underline{40}$ 100 $\underline{250}$ 625

8. a) The perfect squares in the Fibonacci sequence are 1 and 144.
   b) The perfect cubes in the Fibonacci sequences are 1 and 8.
   c) Starting the Fibonacci sequence at 0 changes the results of parts (a) and (b) above. The Fibonacci sequence would then have three perfect squares and three perfect cubes since 0 is a perfect square and a perfect cube.
   ($0^2 = 0$ and $0^3 = 0$)

9. a) 1 1 2 3 5 8 13 21 34 55 89 144 233 377 610 987 1597
   b) Every third term in the sequence is an even number.
   c) Every fifth term in the sequence is a number that is divisible by 5.

10. a) arithmetic: 6 $\underline{30}$ 54 geometric: 6 $\underline{18}$ 54
    b) arithmetic: 6 $\underline{20}$ $\underline{34}$ 48 geometric: 6 $\underline{12}$ $\underline{24}$ 48

11. a) The common difference for this sequence is 40. $\frac{400 - 240}{4} = 40$
    b) 240 $\underline{280}$ $\underline{320}$ $\underline{360}$ 400

12. a) 10 $\underline{40}$ 70

b) 10 $\underline{30}$ $\underline{50}$ 70

c) 10 $\underline{25}$ $\underline{40}$ $\underline{55}$ 70

d) 10 $\underline{22}$ $\underline{34}$ $\underline{46}$ $\underline{58}$ 70

e) 10 $\underline{20}$ $\underline{30}$ $\underline{40}$ $\underline{50}$ $\underline{60}$ 70

Bonus: The numbers worked out nicely above because the total difference of 60 between 10 and 70 divides evenly by 2, 3, 4, 5, and 6. The next row would not work out nicely since 60 does not divide evenly by 7.

13. a) 10.2, 11.9, 13.6 b) This is an arithmetic sequence.

c) $\begin{cases} a_1 = 3.4 \\ a_n = a_{n-1} + 1.7 \end{cases}$ d) $a_{37} = 64.6$

14. a) $\frac{81}{4}, \frac{243}{16}, \frac{729}{64}$ b) This is a geometric sequence.

c) $\begin{cases} u_1 = 64 \\ u_n = \frac{3}{4} u_{n-1} \end{cases}$ d) $u_7 = \frac{729}{64}$

15. a) 4, 9, 13 b) This is a Fibonacci sequence.

c) $\begin{cases} t_1 = 13 \\ t_2 = -7 \\ t_k = t_{k-1} + t_{k-2} \end{cases}$ d) $t_{11} = 57$

16. a) The test considers the answer to be 11 since the sequence seems to be a Fibonacci sequence. Thus, the number after 8 should be 13, not 11.

b) $\begin{cases} a_1 = 1 \\ a_2 = 2 \\ a_n = a_{n-1} + a_{n-2} \end{cases}$ c) Answers can vary, but need explanations.

17. a) 47, 95, 191

b) This sequence represents mixed recursion.

c) $\begin{cases} b_1 = 2 \\ b_i = 2b_{i-1} + 1 \end{cases}$

d) $b_{17} = 196607$

18. a) $\begin{cases} a_1 = 2 \\ a_n = a_{n-1} + 3 \end{cases}$

b) $\begin{cases} b_1 = 3 \\ b_n = 2b_{n-1} \end{cases}$

c) $\begin{cases} c_1 = 0.1 \\ c_n = 0.1c_{n-1} \end{cases}$

d) $\begin{cases} d_1 = -12 \\ d_n = d_{n-1} + 5 \end{cases}$

e) $\begin{cases} a_1 = 3 \\ a_n = -1a_{n-1} \end{cases}$

f) $\begin{cases} u_1 = 25 \\ u_n = 1.2u_{n-1} \end{cases}$

g) $\begin{cases} a_1 = -2 \\ a_2 = 3 \\ a_n = a_{n-1} + a_{n-2} \end{cases}$

h) $\begin{cases} u_1 = 5 \\ u_n = u_{n-1} + n \end{cases}$

19. A geometric sequence increases when the first term is positive and the common ratio is greater than one, or the first term is negative and the common ratio is between 0 and 1. A geometric sequence decreases when the first term is positive and the common ratio is between 0 and 1, or when the first term is negative and the common ratio is greater than 1. A geometric sequence is neither increasing nor decreasing if the common ratio is negative since the sign of the terms would alternate.

20. a) $\begin{cases} a_1 = 2 \\ a_n = 3a_{n-1} \end{cases}$ $a_{17} = 86093442$

b) $\begin{cases} b_1 = 2 \\ b_n = b_{n-1} + 4 \end{cases}$ $b_{12} = 46$

c) $\begin{cases} c_1 = 2 \\ c_2 = 6 \\ c_n = c_{n-1} + c_{n-2} \end{cases}$ $c_{10} = 246$

d) $a_{23} = 149$

e) $t_9 = 1792$ f) $b_{22} = 39603$ g) $u_{15} = 11957422$

21. a) $t_3 = 14,\ t_4 = 18,\ t_5 = 22$

b) $u_3 = 1,\ u_4 = \frac{1}{3},\ u_5 = \frac{1}{9}$

c) $b_3 = 5,\ b_4 = 14,\ b_5 = 41$

d) $t_3 = 9,\ t_4 = 16,\ t_5 = 25$

e) $a_3 = 40,\ a_4 = 1591,\ a_5 = 2531272$

f) $u_3 = \frac{3}{2},\ u_4 = 2,\ u_5 = \frac{5}{2}$

g) $t_3 = 6,\ t_4 = 10,\ t_5 = 16$

h) $v_3 = 8,\ v_4 = 32,\ v_5 = 256$

i) $t_3 = -11,\ t_4 = -17,\ t_5 = 5$

j) $r_3 = 9,\ r_4 = 1,\ r_5 = 64$

22. a) $\begin{cases} a_1 = 7 \\ a_n = a_{n-1} + n - 1 \end{cases}$

b) $\begin{cases} b_1 = 1 \\ b_n = n\,b_{n-1} \end{cases}$

c) $\begin{cases} c_1 = 2 \\ c_n = 2c_{n-1} - 1 \end{cases}$

d) $\begin{cases} d_1 = 1 \\ d_n = d_{n-1} + 2n - 1 \end{cases}$

Note: for (d), see #21(d)

23. a) fixed point: $a_1 = 3$ b) fixed point: $W_1 = 2$

c) fixed point: $u_1 = -2$ d) fixed point: $t_1 = 0$

e) has no fixed point

25. a) let c_n = the price of the calculators ($) during the *n*th week

$$\begin{cases} c_1 = 80 \\ c_n = .75c_{n-1} \end{cases}$$

$c_2 = 60$

The calculators sell for $60 the second week.

b) $c_5 = 25.3125$

The calculators sell for $25.31 during the fifth week.

c) $c_8 = 10.679$ and $c_9 = 8.009$

The calculators will sell for less than $10 during the 9th week.

26. a)

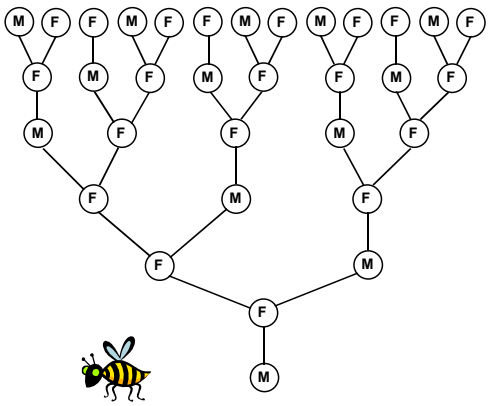
n	0	1	2	3	4	5
R_n	500	560	627.2	702.464	786.760	881.171

b) $$\begin{cases} R_0 = 500 \\ R_n = 1.12 \cdot R_{n-1} \end{cases}$$

c) $R_{14} = 2443.556$ and $R_{15} = 2736.783$

The giant roadrunner population in Arizona will reach at least 2500 in the year 2018.

27. a)



b) 1, 1, 2, 3, 5, 8, 13, 21, … OR 1, 2, 3, 5, 8, 13, 21, …

This is a Fibonacci sequence.

c) let b_n = the number of bees in the nth **previous** generation

Note: This means that the male bee counts as the 0th generation.

$$\begin{cases} b_0 = 1 \\ b_1 = 1 \\ b_n = b_{n-1} + b_{n-2} \end{cases} \quad \text{OR} \quad \begin{cases} b_1 = 1 \\ b_2 = 2 \\ b_n = b_{n-1} + b_{n-2} \end{cases}$$

d) $b_{17} = 2584$

The male bee has 2584 ancestors in the 17th previous generation.

29. a) $\begin{cases} A_0 = 20 \\ A_t = A_{t-1} - 2.3 \end{cases}$ b) $A_4 = 10.8;\ A_8 = 1.6;\ A_9 = -0.7$

The tub contains 10.8 gallons of water after 4 minutes. The tub completely drains after 8/9 minutes.

c) $\begin{cases} B_0 = 20 \\ B_t = 0.75 \cdot B_{t-1} \end{cases}$

d) $A_4 = 6.328;\ A_{10} = 1.1263;\ A_{20} = 0.06342;\ A_{30} = 0.00357$

The tub contains 6.3 gallons of water after 4 minutes. However, using this model, the tub can never completely drain.

e) [can answer either way, provided student gives thorough explanation]

30. a) let W_n = the amount of water in the tub (gallons) after n minutes

$$\begin{cases} W_0 = 20 \\ W_n = 0.75W_{n-1} + 3 \end{cases}$$

b) $W_1 = 18$

The tub will contain 18 gallons of water after 1 minute.

c) $W_6 = 13.424$

The tub will contain 13.4 gallons of water after 6 minutes.

d) consider the table below

n (minutes)	0	10	20	30	40
W_n (gallons)	20	12.451	12.025	12.001	12.00008

In the long run, the amount of water in the tub approaches 12 gallons.

31. a)

n	0	1	2	3	4	5	6
V_n	1	3	7	15	31	63	127

b) $$\begin{cases} V_0 = 1 \\ V_n = 2V_{n-1} + 3 \end{cases}$$

32. a) let C_t = the car's value (\$) t years after purchase

$$\begin{cases} C_0 = 15000 \\ C_t = 1.04C_{t-1} + 25 \end{cases}$$

b) $C_6 = 19145.61$

Six years after your purchase, the car will be worth \$19146.

c) $C_{12} = 24391.13$ and $C_{13} = 25391.77$

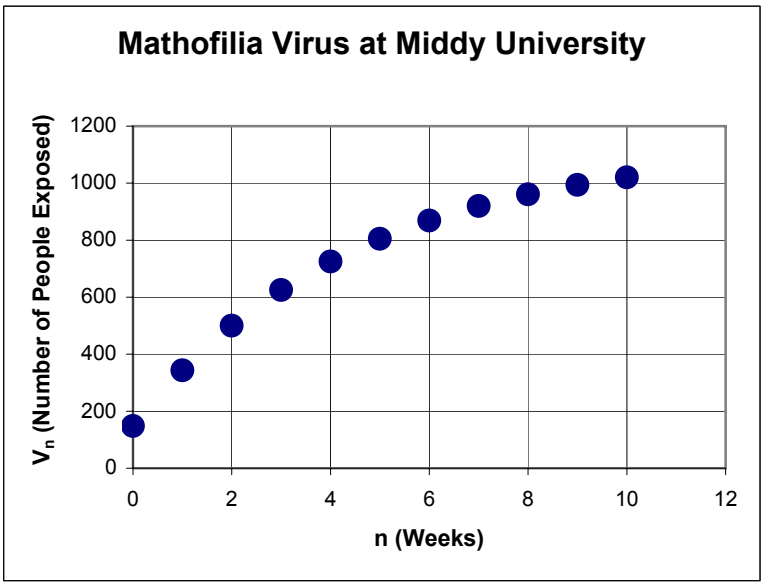
Thirteen years after your purchase, the car will be worth at least \$25,000.

33. a) $\begin{cases} V_2 = 500 \\ V_n = 0.8V_{n-1} + 225 \end{cases}$ OR $\begin{cases} V_0 = 148.4375 \\ V_n = 0.8V_{n-1} + 225 \end{cases}$

b) $V_{10} = 1020.142$

At the end of 10 weeks, 1020 people have been exposed to the mathofilia virus at Middy University.

c)



(Label 2 points!!)

d)

n	V_n
0	148.4375
10	1020.142
20	1113.741
30	1123.791
40	1124.870
50	1124.986
60	1124.999

OR

For a fixed point $V_n = V_{n-1}$

Then $V_n = 0.8V_n + 225$

Let $x = V_n$

$x = 0.8x + 225$

$0.2x = 225$

$x = 1125$

The fixed point of the model is 1125. That is, in the long run, 1125 people at Middy University will have been exposed to the mathofilia virus.