

Read every question carefully and answer the questions based on what you are asked.

Do not leave any question blank and show your work.

You may not leave the exam early; even if you are finished, review the test until the end of the designated time.

Show your work on the paper and make sure you write units.

Please do not ask any questions during the test.

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Question 1: Solve the Following Equations/Inequalities

A) $(3x) - \frac{1}{3}(3x + 3) = \frac{7}{3}$

$$3x - x - 1 = \frac{7}{3}$$
$$\Rightarrow x = \frac{5}{3} = 1.67$$

B) $\frac{4x+4}{-2} = -4x+6$

$$4x + 4 = -2(-4x + 6)$$
$$4x + 4 = 8x - 12$$
$$\Rightarrow x = 4$$

C) $\frac{36x-1}{-3} - 2 = 10x$

$$36x - 1 = -3(10x + 2)$$
$$36x - 1 = -30x - 6$$
$$\Rightarrow x = \frac{-5}{66} = -0.076$$

D) $\frac{36x-1}{-3} = \frac{4x-7}{2}$

$$2(36x - 1) = -3(4x - 7)$$
$$72x - 2 = -12x + 21$$
$$\Rightarrow x = \frac{23}{84} = 0.27$$

$$E) \frac{bcx - d}{-3} - 2 = 10$$

$$\begin{aligned} bcx - d &= -3 \times 12 \\ bcx - d &= -36 \\ bcx &= -36 + d \\ x &= \frac{-36 + d}{bc} \end{aligned}$$

$$F) \frac{3vt}{-3x - 1} - 2 = 1$$

$$\begin{aligned} 3vt &= 3(-3x - 1) \\ 3vt &= -9x - 3 \\ 3vt + 3 &= -9x \\ \frac{3vt + 3}{-9} &= x = \frac{vt + 1}{-3} \text{ after factorization.} \end{aligned}$$

Question 2:

- A) A father is now 5 times older than his son. In 15 years, he will be only twice as old as his son. How old are they now?

$$\begin{aligned} &\begin{cases} \text{son: } x \\ \text{father: } 5x \end{cases} \\ &\begin{cases} \text{son: } x + 15 \\ \text{father: } 5x + 15 \end{cases} \\ 5x + 15 &= 2(x + 15) \Rightarrow x = 5 \\ \text{son} &= 5 \text{ years old, father} = 25 \text{ years old} \end{aligned}$$

- B) Sarah's mom is 42 years old; her aunt is 48 years old. How many years ago was Sarah's aunt 3 times older than her mom?

$$\begin{aligned} 48 - x &= 3(42 - x) \\ 48 - x &= 126 - 3x \Rightarrow x = 39 \\ &39 \text{ years ago} \end{aligned}$$

- C) A cyclist traveling at 20Km/h leaves 30 minutes ahead of a car traveling at 50Km/h . If they go in the same direction, how long will it take for the car to take over the cyclist?

$$\begin{array}{l} \text{Car:} \quad t \quad 50 \quad 50t \\ \text{Cyclist:} \quad t + 0.5 \quad 20 \quad 20(t + 0.5) \end{array}$$

$$50t = 20(t + 0.5) \Rightarrow t = \frac{1}{3}$$

$$\frac{1}{3} \text{ hr or } 20 \text{ min}$$

- D) Of 4 consecutive odd integers, the sum of the 2nd and 4th numbers is equal to 6 more than 2 times the sum of 1st and 3rd numbers; what is the *first* number?

$$\begin{array}{l} \text{Let the integers be: } x, x + 2, x + 4, x + 6 \\ x + 2 + x + 4 = 2(x + x + 4) + 6 \\ \Rightarrow x = -3 \end{array}$$

- E) In a triangle, two of the angles are in a ratio of 5:3. The third angle is 40 degrees less than the smaller of the two. Find the measure of the larger angle:

$$\begin{array}{l} 5x + 3x + 3x - 40 = 180 \\ \Rightarrow x = 20 \text{ degrees} \\ \text{larger angle} = 5x = 100 \text{ degrees} \end{array}$$

- F) Suppose of the two edges (sides) of a rectangle, one is 8 less than 3 times that of the other one. If the perimeter of the rectangle is 24 meters, what is the area of the rectangle?

$$\begin{array}{l} x + 3x - 8 + x + 3x - 8 = 24 \\ \Rightarrow x = 5\text{m}, 3x - 8 = 7\text{m} \\ \Rightarrow \text{Area} = 5 \times 7 = 35 \text{ square meters} \end{array}$$

Type equation here.

- G) A coin collector had a collection of silver coins worth \$205. There were five times as many quarters as half-dollars (50-cent pieces) and 200 fewer dimes than quarters. How many of each coin did the collector have?

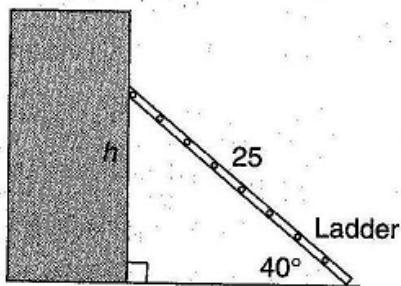
Let half dollars be x , quarters be $5x$, and dimes be $5x - 200$

$$50x + 25(5x) + 10(5x - 200) = 20,500$$

$$\Rightarrow x = 100, \quad 5x = 500, \quad 5x - 200 = 300$$

Question 3:

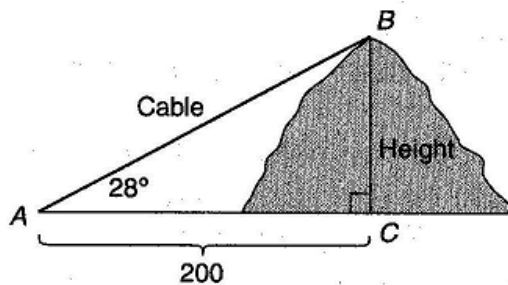
- A) The warning on the side of a 25-ft ladder says that the angle it makes with the ground should be no smaller than 40 degrees before it becomes unsafe. To the nearest foot, what is the height along a wall that this ladder makes at such angle?



$$\sin 40 = \frac{h}{25}$$

$$\Rightarrow h = 16 \text{ ft}$$

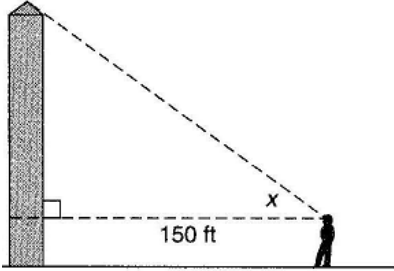
- B) From a point 200 meters from the center of the base of a hill, the angle of elevation to the top of the hill is 28 degrees. A tram from that point carries people to the top. To the nearest meter, how high is the hill, and how long is the tram's cable?



$$\tan 28 = \frac{h}{200} \Rightarrow h = 200 \tan 28 = 106m$$

$$\cos 28 = \frac{200}{c} \Rightarrow c = \frac{200}{\cos 28} = 227m$$

- C) A young tourist visiting D.C. is awed by the size of the Washington Monument, which is 555 ft tall. The tourist, who is 5 ft tall, is standing 150 ft from the monument. At what angle, to the nearest degree, must she elevate her head to look at the top of the monument?



$$555 - 5 = 550 \text{ height from the tourist's eyes}$$
$$\tan x = \frac{550}{150} \Rightarrow x = \tan^{-1} \frac{550}{150} = 74.7 \text{ degrees}$$

- D) If the base of the 25-ft ladder in question 3-A was 15 meters away from the wall, what would the angle of elevation be (instead of 40 degrees)

$$\cos \theta = \frac{15}{25} \Rightarrow \theta = \cos^{-1} \frac{15}{25} = 53.1 \text{ degrees}$$

Question 4:

$$\text{A) } -3 \left(\begin{bmatrix} -2 & 3 & -1 \\ 1 & 0 & 1 \\ -2 & 0 & -3 \end{bmatrix} - \begin{bmatrix} -1 & -2 & -1 \\ -3 & 1 & -1 \\ -2 & 2 & -3 \end{bmatrix} \right) =$$

$$= -3 \begin{bmatrix} -1 & 5 & 0 \\ 4 & -1 & 2 \\ 0 & -2 & 0 \end{bmatrix} = \begin{bmatrix} -3 & -15 & 0 \\ -12 & 3 & -6 \\ 0 & 6 & 0 \end{bmatrix}$$

B) Suppose in a right triangle, the hypotenuse is 100 units and one side is 60 units. Use the Pythagorean theorem to calculate the length of the third side:

$$60^2 + b^2 = 100^2 \Rightarrow b = 80 \text{ units}$$

C) What percent of 50% of 10% of 20 is equal to 25% of 40% of \$30?

$$p \times 0.5 \times 0.1 \times 20 = 0.25 \times 0.4 \times 30 \\ \Rightarrow p = 3 = 300\%$$