More Product, Quotient, Chain Rule (3.6 Day 4)

~~(a), (b), (c) @ end~~

1.) $y' = (x^2-4)^3\left[5(2x^2+5)^4 \cdot 4x\right] + (2x^2+5)^5\left[3(x^2-4)^2 \cdot 2x\right]$

$y' = 20x(x^2-4)^3(2x^2+5)^4 + 6x(2x^2+5)^5(x^2-4)^2$

$y' = 2x(x^2-4)^2(2x^2+5)^4\left[\underbrace{10(x^2-4)}_{10x^2-40} + \underbrace{3(2x^2+5)}_{+6x^2+15}\right]$

$y' = 2x(x^2-4)^2(2x^2+5)^4(16x^2-25)$

$\boxed{\text{y has horizontal tangents @ } x = 0, \pm 2, \pm 5/4}$

#2) $y' = (2x^4+1)(1) + (x-5)(8x^3)$

$y' = 2x^4 + 1 + 8x^4 - 40x^3$

$y' = 10x^4 - 40x^3 + 1$ to have a slope of 1 means $y' = 1$

$1 = 10x^4 - 40x^3 \cancel{+1}$

$0 = 10x^4 - 40x^3$

$0 = 10x^3[x-4]$

ANSWER:
$y+5 = 1(x-0)$
and
$y+513 = 1(x-4)$

so there are two tangents that have a slope of 1 there is a tangent @ $x=0$ and $x=4$
to write the eq. of a tangent I need pt & slope I have $m=1$ & $m=1$
$(0, ?)$ & $(4, ?)$ go back to the original to find the y-coord

$x=0$ & $x=4$
$y=-5$ & $y=-513$

#3.) $y' = \dfrac{(x-4)^3 \cdot [5(x-2)^4 \cdot 1] - (x-2)^5 \cdot [3(x-4)^2 \cdot 1]}{(x-4)^6}$

$y' = \dfrac{5(x-4)^3(x-2)^4 - 3(x-2)^5(x-4)^2}{(x-4)^6}$

$y' = \dfrac{\cancel{(x-4)}^2(x-2)^4\,[5(x-4) - 3(x-2)]}{(x-4)^{\cancel{6}\,4}}$

$y' = \dfrac{(x-2)^4[5x-20-3x+6]}{(x-4)^4} = \dfrac{(x-2)^4(2x-14)}{(x-4)^4}$

$\boxed{\text{so } y \text{ has a horizontal tangent @ } x = 2, 7}$

(note: when a fraction $=0$ that means the numerator $=0$ not the denominator b/c when the denominator $=0$ the fraction is undefined)

#4.) very similar to #3 $\boxed{\begin{array}{l} y - \frac{3}{2} = 5(x - \frac{1}{2}) \\ y + \frac{9}{2} = 5(x + \frac{1}{2}) \end{array}}$

#5.) you can do this find pt & slope & then put it together $\boxed{y=2}$

#6.) √ your $\frac{d}{dx}$ did you get $s'(t) = 3(\tan t)^2(\sec^2 t) - 2$

$s'(t) = 3\tan^2 t \sec^2 t - 2$

now use $s'(t)$ to find slope & use $s(t)$ to find pt then put it together. $\boxed{y - (1 - \frac{\pi}{2}) = 4(t - \frac{\pi}{4})}$

#7.) $r'(\theta) = \sec\theta[-\csc\theta\cot\theta] + \csc\theta[\sec\theta\tan\theta]$

$r'(\theta) = -\sec\theta\csc\theta\cot\theta + \csc\theta\sec\theta\tan\theta$

$r'(\theta) = \sec\theta\csc\theta[-\cot\theta + \tan\theta]$

$0 = (\sec\theta)(\csc\theta)(-\cot\theta + \tan\theta)$

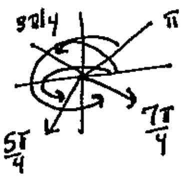
$\sec\theta = 0$ $\quad$ $\csc\theta = 0$ $\quad$ $-\cot\theta + \tan\theta = 0$

$\cancel{\frac{1}{\cos\theta} = 0}$ $\quad$ $\cancel{\frac{1}{\sin\theta} = 0}$ $\quad$ $\tan\theta = \cot\theta$

never b/c $\frac{1}{\text{anything}}$ will never $= 0$

$1 \neq 0$

these are = when θ is $\pi/4$ or @ a ref. ∠ of $\pi/4$ in any quadrant



$\boxed{\text{So } \theta = \frac{\pi}{4} + \frac{k\pi}{2} \text{ where } k \text{ is any integer}}$

#8.) rate of change means $r'(\theta)$; note $r(\theta) = (\sec(2\theta))^4$

$r'(\theta) = 4(\sec 2\theta)^3(\sec 2\theta\tan 2\theta \cdot 2)$

$r'(\theta) = 8\sec^4 2\theta\tan 2\theta$

now find $r'(\frac{\pi}{3})$ → try this on your own you should get a final

$\boxed{r'(\frac{\pi}{3}) = -128\sqrt{3}}$

a.) $f'(x) = \frac{(1-2x)(6x-13)}{(3x+1)^4}$

b.) $g'(x) = 2(x^2-3)^4(2x-1)^2(13x^2-5x-9)$

c.) $h'(x) = \frac{15(1+\sqrt{3x})^4}{2\sqrt{3x}}$