



### Derivative of $\ln x$

We want to find the derivative of:  $y = \ln x$   $\log_e = \ln$

So we need to change it to something we can differentiate:

$$e^y = x$$

We will need to use implicit differentiation:

$$e^y \cdot \frac{dy}{dx} = 1$$

Now we just need to solve:

$$\frac{dy}{dx} = \frac{1}{e^y}$$

And by substitution we get:

$$\frac{dy}{dx} = \frac{1}{x} \cdot 1$$

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Derivative of  $\ln x$ 

If  $u$  is a differentiable function of  $x$  and  $u > 0$ ,

$$\frac{d}{dx} \ln u = \frac{1}{u} \frac{du}{dx}$$

$\frac{d}{dx} \ln u = 1$  over the inside times the derivative of the inside

Find  $\frac{dy}{dx}$  if  $y = \ln(3x^2)$

$$y' = \frac{1}{3x^2} \cdot 6x$$

$$y' = \frac{2}{x}$$

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Derivative of  $\log_a x$ 

We want to find the  $d/dx$  of  $\log_a x$  but we don't know the rule or short cut, but if we recall change of base we can re-write the log as a natural log and then we can differentiate:

$$y = \log_a x$$

change of base formula

$$y = \frac{\ln x}{\ln a} = \frac{1}{\ln a} \cdot \ln x$$

$$y' = \frac{1}{\ln a} \cdot \frac{1}{x} \cdot 1$$

(Note:  $\ln a$  is a constant!)

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Derivative of  $\log_a x$ 

If  $u$  is a differentiable function of  $x$  and  $u > 0$ ,

$$\frac{d}{dx} \log_a u = \frac{1}{u \ln a} \cdot \frac{du}{dx}$$

$\frac{d}{dx} \log_a u = 1$  over (the inside times  $\ln$  of base) times the derivative of inside

Example: Find the rate of change of  $y = \log_3 \sqrt{2x-1}$

$$y = \log_3 (2x-1)^{1/2}$$

$$y' = \frac{1}{(2x-1)^{1/2} \cdot \ln 3} \cdot \frac{1}{2} (2x-1)^{-1/2} \cdot 2$$

$$y' = \frac{1}{(2x-1)^{1/2} \cdot \ln 3 \cdot (2x-1)^{1/2}} = \frac{1}{(2x-1) \ln 3}$$

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