Example **Derivatives of a Higher Order**

$$\text{Find } \frac{d^2y}{dx^2} \text{ if } \quad x^2 + y^2 = 25$$

$$2x\left(\cancel{\frac{dx}{dx}}\right) + 2y\left(\frac{dy}{dx}\right) = 0$$

$$2y\left(\frac{dy}{dx}\right) = -2x$$

$$\frac{dy}{dx} = -\frac{2x}{2y} = -\frac{x}{y}$$

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Example **Derivatives of a Higher Order**

$$\frac{dy}{dx} = -\frac{x}{y}$$

Substitute

$$\frac{d^2y}{dx^2} = \frac{y(-\cancel{\frac{dx}{dx}}) - (-x)(\frac{dy}{dx})}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{\underline{-y} + \underline{(x)(\frac{-x}{y})}}{y^2} \bullet \left(\frac{y}{y}\right)$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{-y^2 - x^2}{y^3}}$$

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Example **Derivatives of a Higher Order**

Remember $x^2 + y^2 = 25$ *and* $\dfrac{d^2y}{dx^2} = \dfrac{-y^2 - x^2}{y^3}$

$$\frac{d^2y}{dx^2} = \frac{-y^2 - x^2}{y^3} = \frac{-(x^2 + y^2)}{y^3}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{-25}{y^3}}$$

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Find all points (both coordinates) on the curve $x^2 - xy + y^2 = 3$ where the tangent line is (a) horizontal and (b) vertical.

$$2x - x\frac{dy}{dx} + y(-1) + 2y\frac{dy}{dx} = 0$$

$$-x\frac{dy}{dx} + 2y\frac{dy}{dx} = y - 2x$$

$$\frac{dy}{dx}\left[-x + 2y\right] = y - 2x$$

$$\boxed{\frac{dy}{dx} = \frac{y - 2x}{-x + 2y}}$$

a.) Horizontal tangent $\Rightarrow \dfrac{dy}{dx} = 0$

Fraction = 0 when numerator = 0

$y - 2x = 0$

$y = 2x$ Substitute into original

$x^2 - x(2x) + (2x)^2 = 3$

$x^2 - 2x^2 + 4x^2 = 3$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

$\boxed{(1, 2)\ (-1, -2)}$

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b.) vertical tangent $\Rightarrow \frac{dy}{dx} = \text{und}$

Fraction is undefined when denominator $= 0$

$$2y - x = 0$$

$$2y = x$$

back into original

$$(2y)^2 - (2y)(y) + y^2 = 3$$

$$3y^2 = 3$$

$$y = \pm 1$$

$$\boxed{(2,1) \text{ \& } (-2,-1)}$$

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