3.8 Day2

Derivatives of Inverse Trigonometric Functions

$y = \sin^{-1}x$

OR

$y = \mathrm{ArcSin}\,x$

Oct 14-10:57 AM

How do we find the derivative of y = $\sin^{-1}(x)$?
Where did this equation come from?
If y = sin x then the inverse is x = sin y which is equivalent to y = $\sin^{-1}x$ so if we find the d/dx of x = siny then that is the derivative we want, so by implicit differentiation...

$1 = \cos y \cdot \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{\cos y}$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-\sin^2 y}}$

$\boxed{\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \cdot 1}$

Recall

$\sin^2 y + \cos^2 y = 1$

$\cos^2 y = 1 - \sin^2 y$

$\cos y = \sqrt{1-\sin^2 y}$

This shows where the rule for #17 in your d/dx formula packet comes from we are not going to derive the other 2 we are simply going to use the shortcuts in the packet from now on!

Oct 21-7:57 AM

Derivative of Inverse Trig Functions

If u is a differentiable function of x with $|u|<1$, we apply the Chain Rule to get

$$\frac{d}{dx}\sin^{-1}u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}, \quad |u|<1.$$

The derivative is defined for all real numbers.
If u is a differentiable function of x, we apply the Chain Rule to get

$$\frac{d}{dx}\tan^{-1}u = \frac{1}{1+u^2}\frac{du}{dx}.$$

If u is a differentiable function of x with $|u|>1$, we have the formula

$$\frac{d}{dx}\sec^{-1}u = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}, \quad |u|>1.$$

Note: the derivative rules for the remaining 3 Inverse Trig functions are the same just negative.

Note:
$\sin^{-1}(x)$ = Arcsin(x)
$\tan^{-1}(x)$=Arctan(x)
and so on for the remaining inverse trig functions.

Oct 14-10:58 AM

We are going to fill in examples on your Derivative Formulas Packet #'s 17 - 22:

If $y=\sin^{-1}8x^2$, find $\frac{dy}{dx}$. (#17 in packet)

$u=8x^2$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-64x^4}} \cdot 16x = \boxed{\frac{16x}{\sqrt{1-64x^4}}}$

If y = $\cos^{-1}(4-x)$, find y'. (#18 in packet)

$u=4-x$

$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(4-x)^2}} \cdot (-1) = \boxed{\frac{1}{\sqrt{1-(4-x)^2}}}$

Oct 14-11:01 AM

If y = $\tan^{-1}(3x)$, find y'. (#19 in packet)

$u=3x$

$y' = \frac{1}{1+9x^2} \cdot 3$

$\boxed{y' = \frac{3}{1+9x^2}}$

If y = $\cot^{-1}(-x^2)$, find y'. (#20 in packet)

$u=-x^2$

$y' = \frac{-1}{1+x^4}(-2x)$

$\boxed{y' = \frac{2x}{1+x^4}}$

Oct 14-11:06 AM

Given $y=\sec^{-1}(3x-4)$, find $\frac{dy}{dx}$. (#21 in packet)

$u=3x-4$

$\boxed{\frac{dy}{dx} = \frac{3}{|3x-4|\sqrt{(3x-4)^2-1}}}$

If y = $\csc^{-1}(x^2+2)$, find dy/dx. (#22 in packet)

$u=x^2+2$

$\frac{dy}{dx} = \frac{-1}{|x^2+2|\sqrt{(x^2+2)^2-1}} \cdot 2x = \boxed{\frac{-2x}{(x^2+2)\sqrt{(x^2+2)^2-1}}}$

Oct 14-11:06 AM