26. $x^2\cos^2 y - \sin y = 0$

$$\frac{d}{dx}(x^2\cos^2 y) - \frac{d}{dx}(\sin y) = \frac{d}{dx}(0)$$

$$(x^2)(2\cos y)(-\sin y)\left(\frac{dy}{dx}\right) + (\cos^2 y)(2x) - (\cos y)\frac{dy}{dx} = 0$$

$$-(2x^2\cos y\sin y + \cos y)\frac{dy}{dx} = -2x\cos^2 y$$

$$\frac{dy}{dx} = \frac{2x\cos^2 y}{\cos y + 2x^2\cos y\sin y} = \frac{2x\cos y}{1+2x^2\sin y}$$

Slope at $(0, \pi)$: $\dfrac{2(0)\cos\pi}{1+2(0)^2\sin\pi} = 0$

(a) Tangent: $y = \pi$

(b) Normal: $x = 0$

27.

$$x^2 + y^2 = 1$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(1)$$

$$2x + 2yy' = 0$$

$$2yy' = -2x$$

$$y' = -\frac{x}{y}$$

$$y'' = \frac{d}{dx}\left(-\frac{x}{y}\right) = -\frac{(y)(1)-(x)(y')}{y^2} = -\frac{y - x\left(-\frac{x}{y}\right)}{y^2} = -\frac{x^2+y^2}{y^3}$$

Since our original equation was $x^2 + y^2 = 1$, we may substitute 1 for $x^2 + y^2$, giving $y'' = -\dfrac{1}{y^3}$.

28.

$$x^{2/3} + y^{2/3} = 1$$

$$\frac{d}{dx}(x^{2/3}) + \frac{d}{dx}(y^{2/3}) = \frac{d}{dx}(1)$$

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}y' = 0$$

$$y' = -\frac{x^{-1/3}}{y^{-1/3}} = -\left(\frac{y}{x}\right)^{1/3}$$

$$y'' = \frac{d}{dx}\left[-\left(\frac{y}{x}\right)^{1/3}\right]$$

$$= -\frac{1}{3}\left(\frac{y}{x}\right)^{-2/3}\frac{d}{dx}\left(\frac{y}{x}\right)$$

$$= -\frac{1}{3}\left(\frac{y}{x}\right)^{-2/3}\frac{xy'-(y)(1)}{x^2}$$

$$= -\frac{1}{3}\,\frac{-(x)\left(\frac{y}{x}\right)^{1/3} - y}{x^{4/3}y^{2/3}}$$

$$= \frac{1}{3}\,\frac{x^{2/3}y^{1/3}+y}{x^{4/3}y^{2/3}}$$

$$= \frac{x^{2/3}+y^{2/3}}{3x^{4/3}y^{1/3}}$$

Since our original equation was $x^{2/3} + y^{2/3} = 1$, we may substitute 1 for $x^{2/3} + y^{2/3}$, giving $y'' = \dfrac{1}{3x^{4/3}y^{1/3}}$.

29.

$$y^2 = x^2 + 2x$$

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(x^2) + \frac{d}{dx}(2x)$$

$$2yy' = 2x + 2$$

$$y' = \frac{2x+2}{2y} = \frac{x+1}{y}$$

$$y'' = \frac{d}{dx}\left(\frac{x+1}{y}\right) = \frac{(y)(1)-(x+1)y'}{y^2} = \frac{y-(x+1)\left(\frac{x+1}{y}\right)}{y^2} = \frac{y^2-(x+1)^2}{y^3}$$

Since our original equation was $y^2 = x^2 + 2x$, we may write $y^2 - (x+1)^2 = (x^2+2x) - (x^2+2x+1) = -1$, which gives $y = -\dfrac{1}{y^3}$.

30. $y^2 + 2y = 2x + 1$

$$\frac{d}{dx}(y^2 + 2y) = \frac{d}{dx}(2x+1)$$

$$(2y+2)y' = 2$$

$$y' = \frac{1}{y+1}$$

30. Continued

$$y'' = \frac{d}{dx}\frac{1}{y+1}$$
$$= -(y+1)^{-2}y'$$
$$= -(y+1)^{-2}\left(\frac{1}{y+1}\right)$$
$$= -\frac{1}{(y+1)^3}$$

31. $\frac{dy}{dx} = \frac{d}{dx}x^{9/4} = \frac{9}{4}x^{(9/4)-1} = \frac{9}{4}x^{5/4}$

32. $\frac{dy}{dx} = \frac{d}{dx}x^{-3/5} = -\frac{3}{5}x^{(-3/5)-1} = -\frac{3}{5}x^{-8/5}$

33. $\frac{dy}{dx} = \frac{d}{dx}\sqrt[3]{x} = \frac{d}{dx}x^{1/3} = \frac{1}{3}x^{(1/3)-1} = \frac{1}{3}x^{-2/3}$

34. $\frac{dy}{dx} = \frac{d}{dx}\sqrt[4]{x} = \frac{d}{dx}x^{1/4} = \frac{1}{4}x^{(1/4)-1} = \frac{1}{4}x^{-3/4}$

35. $\frac{dy}{dx} = \frac{d}{dx}(2x+5)^{-1/2} = -\frac{1}{2}(2x+5)^{(-1/2)-1}\frac{d}{dx}(2x+5)$

$$= -\frac{1}{2}(2x+5)^{-3/2}(2) = -(2x+5)^{-3/2}$$

36. $\frac{dy}{dx} = \frac{d}{dx}(1-6x)^{2/3}$

$$= \frac{2}{3}(1-6x)^{(2/3)-1}\frac{d}{dx}(1-6x)$$
$$= \frac{2}{3}(1-6x)^{-1/3}(-6)$$
$$= -4(1-6x)^{-1/3}$$

37. $\frac{dy}{dx} = \frac{d}{dx}\left(x\sqrt{x^2+1}\right)$

$$= x\frac{d}{dx}\sqrt{x^2+1} + \sqrt{x^2+1}\frac{d}{dx}(x)$$
$$= x\frac{d}{dx}(x^2+1)^{1/2} + (x^2+1)^{1/2}$$
$$= x \cdot \frac{1}{2}(x^2+1)^{-1/2}(2x) + (x^2+1)^{1/2}$$
$$= x^2(x^2+1)^{-1/2} + (x^2+1)^{1/2}$$

Note: This answer is equivalent to $\frac{2x^2+1}{\sqrt{x^2+1}}$.

38. $\frac{dy}{dx} = \frac{d}{dx}\frac{x}{\sqrt{x^2+1}} = \frac{(x^2+1)^{1/2}\frac{d}{dx}x - x\frac{d}{dx}(x^2+1)^{1/2}}{x^2+1}$

$$= \frac{(x^2+1)^{1/2} - x \cdot \frac{1}{2}(x^2+1)^{-1/2}(2x)}{x^2+1}$$
$$= \frac{x^2+1-x^2}{(x^2+1)(x^2+1)^{1/2}}$$
$$= \frac{1}{(x^2+1)^{3/2}}$$
$$= (x^2+1)^{-3/2}$$

39. $\frac{dy}{dx} = \frac{d}{dx}(1-x^{1/2})^{1/2}$

$$= \frac{1}{2}(1-x^{1/2})^{-1/2}\frac{d}{dx}(1-x^{1/2})$$
$$= \frac{1}{2}(1-x^{1/2})^{-1/2}\left(-\frac{1}{2}x^{-1/2}\right)$$
$$= -\frac{1}{4}(1-x^{1/2})^{-1/2}x^{-1/2}$$

40. $\frac{dy}{dx} = \frac{d}{dx}3(2x^{-1/2}+1)^{-1/3}$

$$= -(2x^{-1/2}+1)^{-4/3}\frac{d}{dx}(2x^{-1/2}+1)$$
$$= -(2x^{-1/2}+1)^{-4/3}(-x^{-3/2})$$
$$= x^{-3/2}(2x^{-1/2}+1)^{-4/3}$$

41. $\frac{dy}{dx} = \frac{d}{dx}3(\csc x)^{3/2}$

$$= \frac{9}{2}(\csc x)^{1/2}\frac{d}{dx}(\csc x)$$
$$= \frac{9}{2}(\csc x)^{1/2}(-\csc x \cot x)$$
$$= -\frac{9}{2}(\csc x)^{3/2}\cot x$$

42. $\frac{dy}{dx} = \frac{d}{dx}[\sin(x+5)]^{5/4}$

$$= \frac{5}{4}[\sin(x+5)]^{1/4}\frac{d}{dx}\sin(x+5)$$
$$= \frac{5}{4}[\sin(x+5)]^{1/4}\cos(x+5)$$

43. (a) If $f(x) = \frac{3}{2}x^{2/3} - 3$, then

$f'(x) = x^{-1/3}$ and $f''(x) = -\frac{1}{3}x^{-4/3}$

which contradicts the given equation $f''(x) = x^{-1/3}$.

(b) If $f(x) = \frac{9}{10}x^{5/3} - 7$, then

$f'(x) = \frac{3}{2}x^{2/3}$ and $f''(x) = x^{-1/3}$, which matches the given equation.

(c) Differentiating both sides of the given equation

$f''(x) = x^{-1/3}$ gives $f'''(x) = -\frac{1}{3}x^{-4/3}$, so it *must* be true that $f'''(x) = -\frac{1}{3}x^{-4/3}$.

(d) If $f'(x) = \frac{3}{2}x^{2/3} + 6$, then $f''(x) = x^{-1/3}$, which matches the given equation.

Conclusion: (b), (c), and (d) could be true.

44. **(a)** If $g'(t)=4\sqrt[4]{t}-4$, then

$g''(t)=\frac{d}{dx}(4t^{1/4}-4)=t^{-3/4}=\frac{1}{t^{3/4}}$, which matches the given equation.

(b) Differentiating both sides of the given equation

$g''(t)=\frac{1}{t^{3/4}}=t^{-3/4}$ gives $g'''(t)=-\frac{3}{4}t^{-7/4}$, which is not consistent with $g'''(t)=-\frac{4}{\sqrt[4]{t}}$.

(c) If $g(t)=t-7+\frac{16}{5}t^{5/4}$, then

$g'(t)=1+4t^{1/4}$ and $g''(t)=t^{-3/4}=\frac{1}{t^{3/4}}$, which matches the given equation.

(d) If $g'(t)=\frac{1}{4}t^{1/4}$, then $g''(t)=\frac{1}{16}t^{-3/4}$, which contradicts the given equation.

Conclusion: (a) and (c) could be true.

45. **(a)**

$$y^4=y^2-x^2$$

$$\frac{d}{dx}(y^4)=\frac{d}{dx}(y^2)-\frac{d}{dx}x^2$$

$$4y^3\frac{dy}{dx}=2y\frac{dy}{dx}-2x$$

$$(4y^3-2y)\frac{dy}{dx}=-2x$$

$$\frac{dy}{dx}=\frac{-2x}{4y^3-2y}=\frac{x}{y-2y^3}$$

At $\left(\frac{\sqrt{3}}{4},\frac{\sqrt{3}}{2}\right)$:

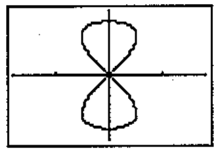
$$\text{Slope}=\frac{\frac{\sqrt{3}}{4}}{\frac{\sqrt{3}}{2}-2\left(\frac{\sqrt{3}}{2}\right)^3}$$

$$=\frac{\frac{\sqrt{3}}{4}}{\frac{\sqrt{3}}{2}-\frac{3\sqrt{3}}{4}}\cdot\frac{\frac{4}{\sqrt{3}}}{\frac{4}{\sqrt{3}}}=\frac{1}{2-3}=-1$$

At $\left(\frac{\sqrt{3}}{4},\frac{1}{2}\right)$:

$$\text{Slope}=\frac{\frac{\sqrt{3}}{4}}{\frac{1}{2}-2\left(\frac{1}{2}\right)^3}=\frac{\frac{\sqrt{3}}{4}}{\frac{1}{2}-\frac{1}{4}}\cdot\frac{4}{4}=\frac{\sqrt{3}}{1}=\sqrt{3}$$

(b)



[−1.8, 1.8] by [−1.2, 1.2]
Parameter interval: $-1\le t\le 1$

46. **(a)**

$$y^2(2-x)=x^3$$

$$\frac{d}{dx}\left[y^2(2-x)\right]=\frac{d}{dx}(x^3)$$

$$(y^2)(-1)+(2-x)(2y)\frac{dy}{dx}=3x^2$$

$$2y(2-x)\frac{dy}{dx}=3x^2+y^2$$

$$\frac{dy}{dx}=\frac{3x^2+y^2}{2y(2-x)}$$

Slope at (1, 1): $\frac{3(1)^2+(1)^2}{2(1)(2-1)}=\frac{4}{2}=2$

Tangent: $y=2(x-1)+1$ or $y=2x-1$

Normal: $y=-\frac{1}{2}(x-1)+1$ or $y=-\frac{1}{2}x+\frac{3}{2}$

(b) One way is to graph the equations $y=\pm\sqrt{\frac{x^3}{2-x}}$.

47. **(a)** $(-1)^3(1)^2=\cos(\pi)$ is true since both sides equal -1.

(b)

$$x^3y^2=\cos(\pi y)$$

$$\frac{d}{dx}(x^3y^2)=\frac{d}{dx}\cos(\pi y)$$

$$(x^3)(2y)\frac{dy}{dx}+(y^2)(3x^2)=(-\sin\pi y)(\pi)\frac{dy}{dx}$$

$$(2x^3y+\pi\sin\pi y)\frac{dy}{dx}=-3x^2y^2$$

$$\frac{dy}{dx}=-\frac{3x^2y^2}{2x^3y+\pi\sin\pi y}$$

Slope at $(-1,1)$: $-\frac{3(-1)^2(1)}{2(-1)^3(1)+\pi\sin\pi}=\frac{-3}{-2}=\frac{3}{2}$

The slope of the tangent line is $\frac{3}{2}$.

48. **(a)** When $x=2$, we have $y^3-2y=-1$, or $y^3-2y+1=0$. Clearly, $y=1$ is one solution, and we may factor y^3-2y+1 as $(y-1)(y^2+y-1)$. The solutions of $y^2+y-1=0$ are $y=\frac{-1\pm\sqrt{(1)^2-4(1)(-1)}}{2(1)}=\frac{-1\pm\sqrt{5}}{2}$.

Hence, there are three possible y-values: 1, $\frac{-1-\sqrt{5}}{2}$, and $\frac{-1+\sqrt{5}}{2}$.

48. Continued

(b)

$$y^3 - xy = -1$$

$$\frac{d}{dx}(y^3) - \frac{d}{dx}(xy) = \frac{d}{dx}(-1)$$

$$3y^2y' - xy' - (y)(1) = 0$$

$$(3y^2 - x)y' = y$$

$$y' = \frac{y}{3y^2 - x}$$

$$y'' = \frac{d}{dx}\frac{y}{3y^2 - x}$$

$$= \frac{(3y^2 - x)(y') - (y)(6yy' - 1)}{(3y^2 - x)^2}$$

$$= \frac{y - xy' - 3y^2y'}{(3y^2 - x)^2}$$

Since we are working with numerical information, there is no need to write a general expression for y'' in terms of x and y.

To evaluate $f'(2)$, evaluate the expression for y' using $x = 2$ and $y = 1$:

$$f'(2) = \frac{1}{3(1)^2 - 2} = \frac{1}{1} = 1$$

To evaluate $f''(2)$, evaluate the expression for y'' using $x = 2$, $y = 1$, and $y' = 1$:

$$f''(2) = \frac{(1) - 2(1) - 3(1)^2(1)}{[3(1)^2 - 2]^2} = \frac{-4}{1} = -4$$

49. Find the two points:

The curve crosses the x-axis when $y = 0$, so the equation becomes $x^2 + 0x + 0 = 7$, or $x^2 = 7$. The solutions are $x = \pm\sqrt{7}$, so the points are $(\pm\sqrt{7}, 0)$.

Show tangents are parallel:

$$x^2 + xy + y^2 = 7$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(xy) + \frac{d}{dx}(y^2) = \frac{d}{dx}(7)$$

$$2x + x\frac{dy}{dx} + (y)(1) + 2y\frac{dy}{dx} = 0$$

$$(x + 2y)\frac{dy}{dx} = -(2x + y)$$

$$\frac{dy}{dx} = -\frac{2x + y}{x + 2y}$$

Slope at $(\sqrt{7}, 0)$: $-\dfrac{2\sqrt{7}) + 0}{\sqrt{7} + 2(0)} = -2$

Slope at $(-\sqrt{7}, 0)$: $-\dfrac{2(-\sqrt{7}) + 0}{-\sqrt{7} + 2(0)} = -2$

The tangents at these points are parallel because they have the same slope. The common slope is -2.

50.

$$x^2 + xy + y^2 = 7$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(xy) + \frac{d}{dx}(y^2) = \frac{d}{dx}(7)$$

$$2x + x\frac{dy}{dx} + (y)(1) + 2y\frac{dy}{dx} = 0$$

$$(x + 2y)\frac{dy}{dx} = -(2x + y)$$

$$\frac{dy}{dx} = -\frac{2x + y}{x + 2y}$$

(a) The tangent is parallel to the x-axis when $\frac{dy}{dx} = -\frac{2x + y}{x + 2y} = 0$, or $y = -2x$.

Substituting $-2x$ for y in the original equation, we have

$$x^2 + xy + y^2 = 7$$

$$x^2 + (x)(-2x) + (-2x)^2 = 7$$

$$x^2 - 2x^2 + 4x^2 = 7$$

$$3x^2 = 7$$

$$x = \pm\sqrt{\frac{7}{3}}$$

The points are $\left(-\sqrt{\frac{7}{3}}, 2\sqrt{\frac{7}{3}}\right)$ and $\left(\sqrt{\frac{7}{3}}, -2\sqrt{\frac{7}{3}}\right)$.

(b) Since x and y are interchangeable in the original equation, $\frac{dx}{dy}$ can be obtained by interchanging x and y in the expression for $\frac{dy}{dx}$. That is, $\frac{dx}{dy} = -\frac{2y + x}{x + 2y}$. The tangent is parallel to the y-axis when $\frac{dx}{dy} = 0$, or $x = -2y$. Substituting $-2y$ for x in the original equation, we have:

$$x^2 + xy + y^2 = 7$$

$$(-2y)^2 + (-2y)(y) + y^2 = 7$$

$$4y^2 - 2y^2 + y^2 = 7$$

$$3y^2 = 7$$

$$y = \pm\sqrt{\frac{7}{3}}$$

The points are $\left(-2\sqrt{\frac{7}{3}}, \sqrt{\frac{7}{3}}\right)$ and $\left(2\sqrt{\frac{7}{3}}, -\sqrt{\frac{7}{3}}\right)$.

Note that these are the same points that would be obtained by interchanging x and y in the solution to part (a).

51. First curve:

$$2x^2 + 3y^2 = 5$$

$$\frac{d}{dx}(2x^2) + \frac{d}{dx}(3y^2) = \frac{d}{dx}(5)$$

$$4x + 6y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{4x}{6y} = -\frac{2x}{3y}$$

51. Continued

Second curve:

$$y^2 = x^3$$
$$\frac{d}{dx}y^2 = \frac{d}{dx}x^3$$
$$2y\frac{dy}{dx} = 3x^2$$
$$\frac{dy}{dx} = \frac{3x^2}{2y}$$

At (1, 1), the slopes are $-\frac{2}{3}$ and $\frac{3}{2}$ respectively.

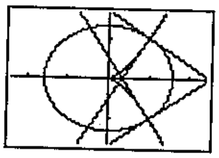
At (1, −1), the slopes are $\frac{2}{3}$ and $-\frac{3}{2}$ respectively. In both cases, the tangents are perpendicular. To graph the curves and normal lines, we may use the following parametric equations for $-\pi \le t \le \pi$:

First curve: $x = \sqrt{\frac{5}{2}}\cos t,\ y = \sqrt{\frac{5}{3}}\sin t$

Second curve: $x = \sqrt[3]{t^2},\ y = t$

Tangents at (1, 1): $x = 1+3t,\ y = 1-2t$
$x = 1+2t,\ y = 1+3t$

Tangents at (1, −1): $x = 1+3t,\ y = -1+2t$
$x = 1+2t,\ y = -1-3t$



[−2.4, 2.4] by [−1.6, 1.6]

52. $v(t) = s'(t) = \frac{d}{dt}(4+6t)^{3/2} = \frac{3}{2}(4+6t)^{1/2}(6)$
$= 9(4+6t)^{1/2}$
$a(t) = v'(t) = \frac{d}{dt}[9(4+6t)^{1/2}]$
$= \frac{9}{2}(4+6t)^{-1/2}(6) = 27(4+6t)^{-1/2}$

At $t = 2$, the velocity is $v(2) = 36$ m/sec and the acceleration is $a(2) = \frac{27}{4}\text{ m/sec}^2$.

53. Acceleration $= \frac{dv}{dt} = \frac{d}{dt}[8(s-t)^{1/2}+1]$
$= 4(s-t)^{-1/2}\left(\frac{ds}{dt}-1\right)$
$= 4(s-t)^{-1/2}(v-1)$
$= 4(s-t)^{-1/2}[(8(s-t)^{1/2}+1)-1]$
$= 32(s-t)^{-1/2}(s-t)^{1/2}$
$= 32\text{ ft/sec}^2$

54. $y^4 - 4y^2 = x^4 - 9x^2$

$$\frac{d}{dx}(y^4) - \frac{d}{dx}(4y^2) = \frac{d}{dx}(x^4) - \frac{d}{dx}(9x^2)$$
$$4y^3\frac{dy}{dx} - 8y\frac{dy}{dx} = 4x^3 - 18x$$
$$\frac{dy}{dx} = \frac{4x^3-18x}{4y^3-8y} = \frac{2x^3-9x}{2y^3-4y}$$

Slope at (3,2): $\frac{2(3)^3-9(3)}{2(2)^3-4(2)} = \frac{27}{8}$

Slope at (−3,2): $\frac{2(-3)^3-9(-3)}{2(2)^3-4(2)} = -\frac{27}{8}$

Slope at (−3,−2): $\frac{2(-3)^3-9(-3)}{2(-2)^3-4(-2)} = \frac{27}{8}$

Slope at (3,−2): $\frac{2(3)^3-9(3)}{2(-2)^3-4(-2)} = -\frac{27}{8}$

55. (a) $x^3 + y^3 - 9xy = 0$

$$\frac{d}{dx}(x^3) + \frac{d}{dx}(y^3) - 9\frac{d}{dx}(xy) = \frac{d}{dx}(0)$$
$$3x^2 + 3y^2\frac{dy}{dx} - 9x\frac{dy}{dx} - 9(y)(1) = 0$$
$$(3y^2-9x)\frac{dy}{dx} = 9y - 3x^2$$
$$\frac{dy}{dx} = \frac{9y-3x^2}{3y^2-9x} = \frac{3y-x^2}{y^2-3x}$$

Slope at (4, 2): $\frac{3(2)-(4)^2}{(2)^2-3(4)} = \frac{-10}{-8} = \frac{5}{4}$

Slope at (2, 4): $\frac{3(4)-(2)^2}{(4)^2-3(2)} = \frac{8}{10} = \frac{4}{5}$

(b) The tangent is horizontal when

$\frac{dy}{dx} = \frac{3y-x^2}{y^2-3x} = 0$, or $y = \frac{x^2}{3}$.

Substituting $\frac{x^2}{3}$ for y in the original equation, we have:

$$x^3 + y^3 - 9xy = 0$$
$$x^3 + \left(\frac{x^2}{3}\right)^3 - 9x\left(\frac{x^2}{3}\right) = 0$$
$$x^3 + \frac{x^6}{27} - 3x^3 = 0$$
$$\frac{x^3}{27}(x^3 - 54) = 0$$
$$\therefore\ x = 0 \text{ or } x = \sqrt[3]{54} = 3\sqrt[3]{2}$$

At $x = 0$, we have $y = \frac{0^2}{3} = 0$, which gives the point (0, 0), which is the origin. At $x = 3\sqrt[3]{2}$, we have $y = \frac{1}{3}(3\sqrt[3]{2})^2 = \frac{1}{3}(9\sqrt[3]{4}) = 3\sqrt[3]{4}$, so the point other than the origin is $(3\sqrt[3]{2}, 3\sqrt[3]{4})$ or approximately (3.780, 4.762).