

Special Equations and Solutions

For use with Lesson 1.3

Most linear equations you have seen in algebra have one solution that makes the equation true. Sometimes, however, you will find equations that have no solution or all real numbers as solutions.

KEY CONCEPTS**Equations with no solution**

An equation has no solution if the equation simplifies to $0 = k$, and $k \neq 0$. $0 = k$ is a contradiction. It is never true. So, no value for the variable makes the equation true.

Equations with all real numbers as solutions

An equation has all real numbers for the solution if the equation simplifies to $0 = 0$. $0 = 0$ is an identity. It is always true. So, all real values of the variable make the equation true.

EXAMPLE 1 Identify equations with no solution or all real numbers as solutions

Determine if the equation has no solution or all real numbers as solutions. Explain.

a. $4(3 - x) = 8 - x + 4 - 3x$

b. $5 - 2(x + 1) = 5 - 2x + 1$

Solution:

- a. The solution is all real numbers. $4(3 - x) = 8 - x + 4 - 3x$ simplifies to $12 - 4x = 12 - 4x$. This simplifies further to $0 = 0$. Both sides of the equation are always equal, regardless of the value of x .
- b. The equation has no solution. $5 - 2(x + 1) = 5 - 2x + 1$ simplifies to $3 - 2x = 6 - 2x$. This simplifies to $0 = 3$, which is never true. So no values of x will make both sides of the equation equal. ■

Recall that the set of real numbers includes both rational and irrational numbers. Solutions to linear equations are from the set of real numbers. You have seen equations having rational number solutions like 8, 0, -6 , and $\frac{5}{3}$. Example 2 explores equations having irrational numbers as solutions.

EXAMPLE 2 Solve equations with irrational numbers as solutions

Solve the equation. Check your solution.

a. $x - 4\pi = 1$

b. $2x + \sqrt{7} = 3$

Solution:

a. $x - 4\pi = 1$

$x = 1 + 4\pi$

Write original equation.

Add 4π to each side.

b. $2x + \sqrt{7} = 3$

$2x = 3 - \sqrt{7}$

Write original equation.

Subtract $\sqrt{7}$ from each side.

$x = \frac{3 - \sqrt{7}}{2}$

Divide each side by 2.

Special Equations and Solutions *continued**Use after Lesson 1.3**Check*

$$\begin{aligned} \text{a. } x - 4\pi &= 1 \\ 1 + 4\pi - 4\pi &= 1 \\ 1 &= 1 \end{aligned}$$

Write original equation.

Substitute $1 + 4\pi$ for x .

Solution checks.

$$\begin{aligned} \text{b. } 2x + \sqrt{7} &= 3 \\ 2\left(\frac{3 - \sqrt{7}}{2}\right) + \sqrt{7} &= 3 \\ 3 - \sqrt{7} + \sqrt{7} &= 3 \\ 3 &= 3 \end{aligned}$$

Write original equation.

Substitute $\frac{3 - \sqrt{7}}{2}$ for x .

Multiply.

Solution checks. ■

Notice that these solutions, $x = 1 + 4\pi$ and $x = \frac{3 - \sqrt{7}}{2}$, are irrational numbers. Since solutions to linear equations must be real numbers, it is possible for a solution to be an irrational number as well as a rational number.

Practice

1. Explain how to tell when an equation has no solution or all real numbers as solutions.
2. In the equation $3(1 - 2x) = nx - m - 2x + 4$, what values of m and n make the equation have no solution? What values make the solution all real numbers?

Determine if the equation has no solution or all real numbers as solutions. Explain.

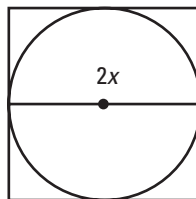
3. $-2(3x - 4) = 6(2 - x)$
4. $3x - (2 - x) = 4(x - 1)$
5. $3x + 5(2 - x) = 4x + 3 - 6x + 7$
6. $8(1 + x) = 4(2x + 2)$

Solve the equation. Check your solution.

7. $\frac{1}{2}x - \frac{2}{3} = x + \frac{3}{4}$
8. $2x - 2\sqrt{3} = 3$
9. $3x - \pi = 5\pi$
10. $3x + e = 9 + 4e$
11. $4 + \pi x = 3\pi$
12. $2\sqrt{2} + x = 3x - 10$

Problem Solving

13. The difference between the perimeters of the square and circle is 5.



Write an equation showing this difference. Then solve the equation. Recall that the perimeter of a circle is $2\pi r$.