

# Mathematics

## Grade Six

By the end of grade six, students have mastered the four arithmetic operations with whole numbers, positive fractions, positive decimals, and positive and negative integers; they accurately compute and solve problems. They apply their knowledge to statistics and probability. Students understand the concepts of mean, median, and mode of data sets and how to calculate the range. They analyze data and sampling processes for possible bias and misleading conclusions; they use addition and multiplication of fractions routinely to calculate the probabilities for compound events. Students conceptually understand and work with ratios and proportions; they compute percentages (e.g., tax, tips, interest). Students know about  $IC$  and the formulas for the circumference and area of a circle. They use letters for numbers in formulas involving geometric shapes and in ratios to represent an unknown part of an expression. They solve one-step linear equations.

## Number Sense

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**Students compare and order positive and negative fractions, decimals, and mixed numbers. Students solve problems involving fractions, ratios, proportions, and percentages:**

- 1.1 Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line.
- 1.3 Use proportions to solve problems (e.g., determine the value of  $N$  if  $4/7 = N/21$ , find the length of a side of a polygon similar to a known polygon). Use cross-multiplication as a method for solving such problems, understanding it as the multiplication of both sides of an equation by a multiplicative inverse.
- 1.4 Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned, and tips.

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**Students calculate and solve problems involving addition, subtraction, multiplication, and division:**

- 2.3 Solve addition, subtraction, multiplication, and division problems, including those arising in concrete situations, that use positive and negative integers and combinations of these operations.
- 2.4 Determine the least common multiple and the greatest common divisor of whole numbers; use them to solve problems with fractions (e.g., to find a common denominator to add two fractions or to find the reduced form for a fraction).

## Algebra and Functions

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**Students write verbal expressions and sentences as algebraic expressions and equations; they evaluate algebraic expressions, solve simple linear equations, and graph and interpret their results:**

1.1 Write and solve one-step linear equations in one variable.

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**Students analyze and use tables, graphs, and rules to solve problems involving rates and proportions:**

2.2 Demonstrate an understanding that *rate* is a measure of one quantity per unit value of another quantity.

## Measurement and Geometry

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**Students deepen their understanding of the measurement of plane and solid shapes and use this understanding to solve problems:**

1.1 Understand the concept of a constant such as  $\pi$ ; know the formulas for the circumference and area of a circle.

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**Students identify and describe the properties of two-dimensional figures:**

2.2 Use the properties of complementary and supplementary angles and the sum of the angles of a triangle to solve problems involving an unknown angle.

# Statistics, Data Analysis, and Probability

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## Students compute and analyze statistical measurements for data sets: 1.1

Compute the range, mean, median, and mode of data sets.

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## Students use data samples of a population and describe the characteristics and limitations of the samples:

2.2 Identify different ways of selecting a sample (e.g., convenience sampling, responses to a survey, random sampling) and which method makes a sample more representative for a population.

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## Students determine theoretical and experimental probabilities and use these to make predictions about events:

3.1 Represent all possible outcomes for compound events in an organized way (e.g., tables, grids, tree diagrams) and express the theoretical probability of each outcome.

3.3 Represent probabilities as ratios, proportions, decimals between 0 and 1, and percentages between 0 and 100 and verify that the probabilities computed are reasonable; know that if  $P$  is the probability of an event,  $1-P$  is the probability of an event not occurring.

# Mathematics

## Grade Seven

By the end of grade seven, students are adept at manipulating numbers and equations and understand the general principles at work. Students understand and use factoring of numerators and denominators and properties of exponents. They know the Pythagorean Theorem and solve problems in which they compute the length of an unknown side. Students know how to compute the surface area and volume of basic three-dimensional objects and understand how area and volume change with a change in scale. Students make conversions between different units of measurement. They know and use different representations of fractional numbers (fractions, decimals, and percents) and are proficient at changing from one to another. They increase their facility with ratio and proportion, compute percents of increase and decrease, and compute simple and compound interest. They graph linear functions and understand the idea of slope and its relation to ratio.

## Number Sense

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**Students know the properties of, and compute with, rational numbers expressed in a variety of forms:**

- 1.2 Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.
- 1.3 Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.
- 1.5 Know that every rational number is either a terminating or repeating decimal and be able to convert terminating decimals into reduced fractions.
- 1.7 Solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest.

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**Students use exponents, powers, and roots and use exponents in working with fractions:**

- 2.3 Multiply, divide, and simplify rational numbers by using exponent rules.
- 2.5 Understand the meaning of the absolute value of a number; interpret the absolute value as the distance of the number from zero on a number line; and determine the absolute value of real numbers.

# Algebra and Functions

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## **Students express quantitative relationships by using algebraic terminology, expressions, equations, inequalities, and graphs:**

- 1.1 Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).
- 1.3 Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used.

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## **Students interpret and evaluate expressions involving integer powers and simple roots:**

- 2.1 Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.

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## **Students graph and interpret linear and some nonlinear functions:**

- 3.3 Graph linear functions, noting that the vertical change (change in y-value) per unit of horizontal change (change in x-value) is always the same and know that the ratio ("rise over run") is called the slope of a graph.
- 3.4 Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the quantities.

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## **Students solve simple linear equations and inequalities over the rational numbers:**

- 4.1 Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.
- 4.2 Solve multistep problems involving rate, average speed, distance, and time or a direct variation.

## Measurement and Geometry

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**Students choose appropriate units of measure and use ratios to convert within and between measurement systems to solve problems:**

- 1.3 Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.

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**Students compute the perimeter, area, and volume of common geometric objects and use the results to find measures of less common objects. They know how perimeter, area, and volume are affected by changes of scale:**

- 2.1 Use formulas routinely for finding the perimeter and area of basic two-dimensional figures and the surface area and volume of basic three-dimensional figures, including rectangles, parallelograms, trapezoids, squares, triangles, circles, prisms, and cylinders.

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**Students know the Pythagorean Theorem and deepen their understanding of plane and solid geometric shapes by constructing figures that meet given conditions and by identifying attributes of figures:**

- 3.3 Know and understand the Pythagorean Theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean Theorem by direct measurement.

## Mathematical Reasoning

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**Students make decisions about how to approach problems:**

- 1.1 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, identifying missing information, sequencing and prioritizing information, and observing patterns.
- 1.2 Formulate and justify mathematical conjectures based on a general description of the mathematical question or problem posed.
- 1.3 Determine when and how to break a problem into simpler parts.

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**Students use strategies, skills, and concepts in finding solutions:**

- 2.1 Use estimation to verify the reasonableness of calculated results.
- 2.2 Apply strategies and results from simpler problems to more complex problems.
- 2.3 Estimate unknown quantities graphically and solve for them by using logical reasoning and arithmetic and algebraic techniques.
- 2.4 Make and test conjectures by using both inductive and deductive reasoning.
- 2.5 Use a variety of methods, such as words, numbers, symbols, charts, graphs, tables, diagrams, and models, to explain mathematical reasoning.
- 2.6 Express the solution clearly and logically by using the appropriate mathematical notation and terms and clear language; support solutions with evidence in both verbal and symbolic work.
- 2.7 Indicate the relative advantages of exact and approximate solutions to problems and give answers to a specified degree of accuracy.
- 2.8 Make precise calculations and check the validity of the results from the context of the problem.

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**Students determine a solution is complete and move beyond a particular problem****by generalizing to other situations:**

- 3.1 Evaluate the reasonableness of the solution in the context of the original situation.
- 3.2 Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.
- 3.3 Develop generalizations of the results obtained and the strategies used and apply them to new problem situations.

# Mathematics

## Grade Eight

### Algebra Readiness

It is imperative for students, whether in grade eight, grade nine, or even a later grade, to master pre-algebraic skills and concepts before they enroll in a course that meets or exceeds the rigor of the content standards for Algebra I adopted by the State Board of Education. The 12 standards that are the target of the algebra readiness program specified in this section are organized into a set of nine topics (this organization is not required for the materials but is included for illustrative purposes.). These standards are purposefully limited in number to provide publishers and teachers the flexibility and time to rebuild foundational skills and concepts that may be missing from earlier grades.

### Number Sense

- 1.2 Add, subtract, multiply and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole number powers.
- 1.5 Know that every rational number is either a terminating or repeating decimal and be able to convert terminating decimals into reduced fractions.

### Algebra and Functions

- 1.3 Simplify numerical expressions by applying properties of rational number (e.g. identity, inverse, distributive, associative, commutative) and justify the process used.
- 3.3 Graph linear functions, noting that the vertical change (change in y-value) per unit of horizontal change (change in x-value) is always the same and know that the ratio (“rise over run”) is called the slope of the graph.
- 3.4 Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the ratio of the quantities.
- 4.1 Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.
- 4.2 Solve multistep problems involving rate, average speed, distance, and time or direct variation.

### Measurement and Geometry

- 1.3 Use measures expressed as rates (e.g. speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.
- 3.3 Know and understand the Pythagorean Theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean Theorem by direct measurement.

## Algebra I

- 2.0 Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root and raising to a fractional power. They understand and use the rules of exponents [excluding fractional powers].
- 4.0 Students simplify expressions before solving linear equations and inequalities in one variable, such as  $3(2x-5) + 4(x-20) = 12$  [excluding inequalities].
- 5.0 Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step [excluding inequalities].

## Algebra I

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**Symbolic reasoning and calculations with symbols are central in algebra. Through the study of algebra, a student develops an understanding of the symbolic language of mathematics and the sciences. In addition, algebraic skills and concepts are developed and used in a wide variety of problem-solving situations.**

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- 2.0 Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents.

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- 4.0 Students simplify expressions before solving linear equations and inequalities in one variable, such as  $3(2x-5) + 4(x-2) = 12$ .

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- 5.0 Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step.

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- 6.0 Students graph a linear equation and compute the x- and y-intercepts (e.g., graph  $2x + 6y = 4$ ). They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by  $2x + 6y < 4$ ).

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- 7.0 Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations by using the point-slope formula.

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- 9.0 Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets.

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- 10.0 Students add, subtract, multiply, and divide monomials and polynomials. Students solve multistep problems, including word problems, by using these techniques.

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- 12.0 Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms.

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- 13.0 Students add, subtract, multiply, and divide rational expressions and functions. Students solve

both computationally and conceptually challenging problems by using these techniques.

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14.0 Students solve a quadratic equation by factoring or completing the square.

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15.0 Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.

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19.0 Students know the quadratic formula and are familiar with its proof by completing the square.

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20.0 Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations.

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21.0 Students graph quadratic functions and know that their roots are the x-intercepts.

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23.0 Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity.