Name: Date:

MPM2D: Quadrilateral Properties

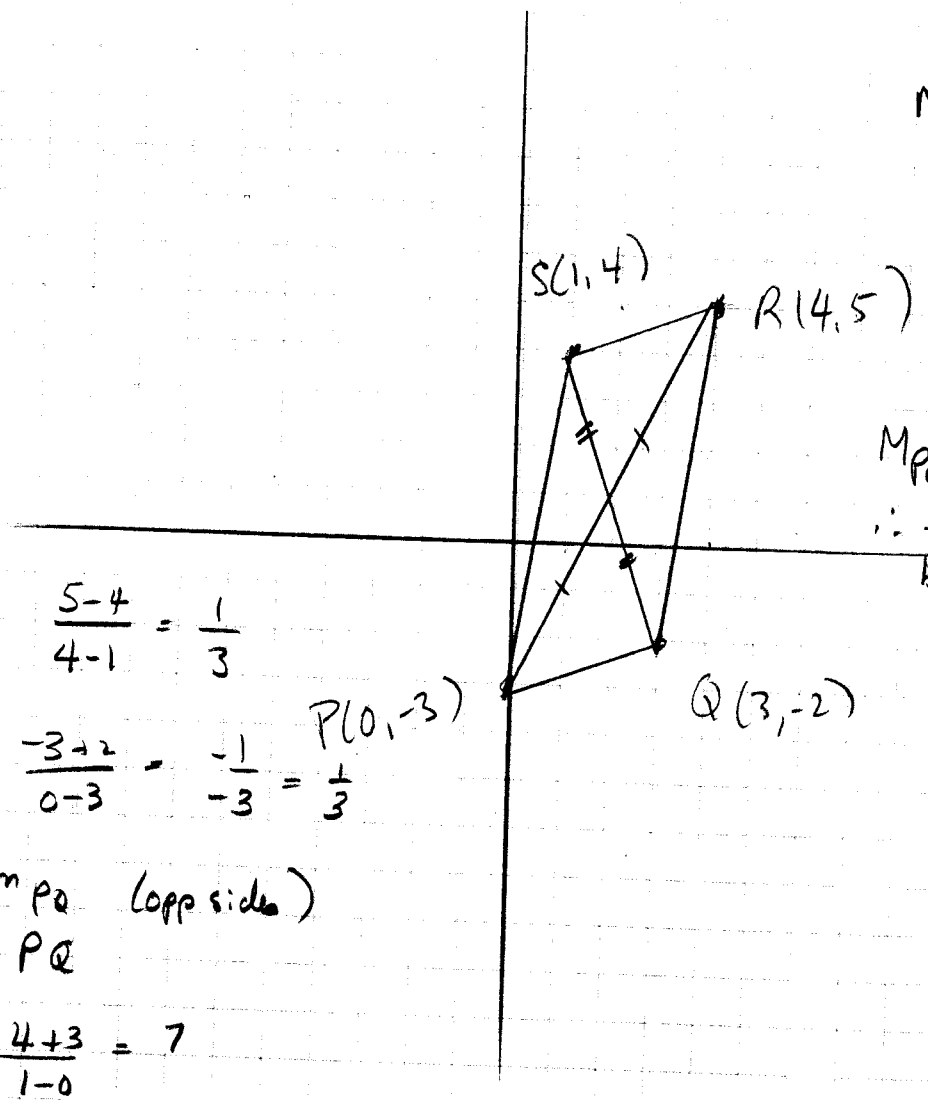
Square Four sides of equal length, four right angles.
Rectangle Opposite sides are equal length, four right angles.
Rhombus Four equal length sides, opposite sides are parallel.
Parallelogram Opposite sides are equal length, opposite sides are parallel.
Trapezoid One pair of parallel sides, not of equal length.
Kite Two pairs of equal length sides, equal sides are not opposite each other.
Irregular All other quadrilaterals.

Exercises

Complete all questions on a separate sheet(s) of paper, and attach it to this sheet. Show all steps.

1. $P(0,-3)$, $Q(3,-2)$, $R(4,5)$ and $S(1,4)$ are the vertices of quadrilateral $PQRS$. Show that the diagonals of $PQRS$ bisect each other. What type of quadrilateral is $PQRS$?
2. $A(4,-1)$, $B(6,5)$, $C(0,11)$ and $D(-6,1)$ are the vertices of quadrilateral $ABCD$. Show that the midpoints of the sides of $ABCD$ form a parallelogram. What type of quadrilateral is $ABCD$?
3. $E(-6,3)$, $F(-5,6)$, $G(-2,7)$ and $H(3,-2)$ are the vertices of quadrilateral $EFGH$. Show that the midpoints of the sides of $EFGH$ form a rectangle. What type of quadrilateral is $EFGH$?
4. Quadrilateral $HJKL$ has vertices $H(5,4)$, $J(-2,5)$, $K(-1,-2)$ and $L(6,-3)$. Show that $HJKL$ is a rhombus.
5. Quadrilateral $ABCD$ has vertices $A(1,5)$, $B(9,6)$, $C(5,-1)$ and $D(-3,-2)$. Show that the diagonals bisect each other at right angles. What type of quadrilateral is $ABCD$?
6. $P(-3,0)$, $Q(-1,5)$ and $R(8,4)$ are three vertices of a parallelogram. Find the coordinates of the fourth vertex S, and describe your procedure for finding it.

#1



$M_{SQ} = \left(\frac{4}{2}, \frac{2}{2}\right)$
$= (2,1)$

$M_{PR} = \left(\frac{4}{2}, \frac{5-3}{2}\right)$
$= (2,1)$

$M_{PR} = M_{SQ}$ ∴ the diagonals do bisect each other.

$m_{RS} = \frac{5-4}{4-1} = \frac{1}{3}$

$m_{PQ} = \frac{-3+2}{0-3} = \frac{-1}{-3} = \frac{1}{3}$

$m_{RS} = m_{PQ}$ (opp sides)
∴ RS ∥ PQ

$m_{SP} = \frac{4+3}{1-0} = 7$

$m_{RQ} = \frac{5+2}{4-1} = 7$

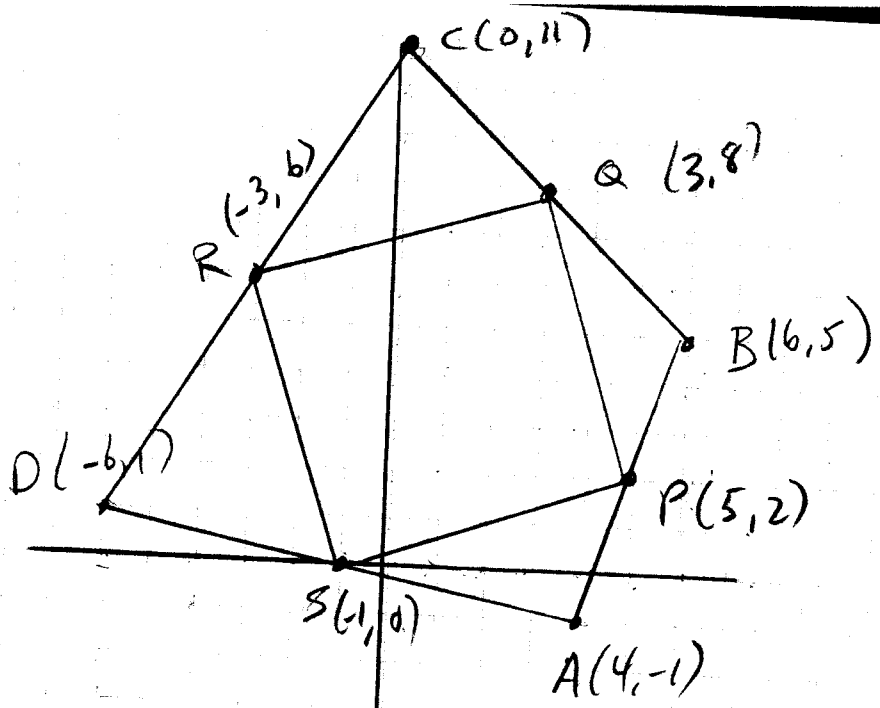
$m_{SP} = m_{RQ}$ (opp sides)
∴ SP ∥ RQ + PQRS is a ∥gm.
but slopes of adjacent sides are not neg. reciprocals.
∴ PQRS ≠ rectangle/square.

$d_{SR} = \sqrt{(5-4)^2 + (4-1)^2}$
$= \sqrt{1+9}$
$= \sqrt{10}$

$d_{SP} = \sqrt{(4+3)^2 + (1)^2}$
$= \sqrt{50}$

∴ lengths of adjacent sides are not equal
∴ PQRS is a parallelogram but not a rhombus, square or rectangle

2.



$m_{RS} = \frac{6-0}{-3+1} = \frac{6}{-2} = -3$

$m_{QP} = \frac{8-2}{3-5} = \frac{6}{-2} = -3$

$m_{RS} = m_{QP}$

$\therefore RS \parallel QP$

$m_{RQ} = \frac{8-6}{3+3} = \frac{2}{6} = \frac{1}{3}$

$m_{SP} = \frac{2-0}{5+1} = \frac{2}{6} = \frac{1}{3}$

$m_{RQ} = m_{SP}$

$\therefore RQ \parallel SP$ + PQRS is a $\parallel$gm.

$M_{AB} = \left(\frac{6+4}{2}, \frac{5-1}{2}\right) = (5,2) = P$

$M_{BC} = \left(\frac{6}{2}, \frac{16}{2}\right) = (3,8) = Q$

$M_{CD} = \left(\frac{-6}{2}, \frac{11+1}{2}\right) = (-3\ 6) = R$

$M_{DA} = \left(\frac{-6+4}{2}, \frac{1-1}{2}\right) = (-1, 0) = S$

What type of Quadrilateral is ABCD?

$m_{AB} = \frac{3}{1} = 3.$

$m_{CD} = \frac{11-1}{+6} = \frac{10}{6} = \frac{5}{3}$

$m_{BC} = \frac{11-5}{0-6} = \frac{6}{-6} = -1$

$m_{AD} = \frac{1+1}{-6-4} = \frac{2}{-10} = -\frac{1}{5}$

No sides are $\parallel$ or at 90°

$\therefore$ ABCD is not type of $\parallel$gm a trapezoid but could be a kite.

$d_{CD} = \sqrt{(10+6)^2 + (11-1)^2}$

$= \sqrt{36+100}$

$= \sqrt{136}$

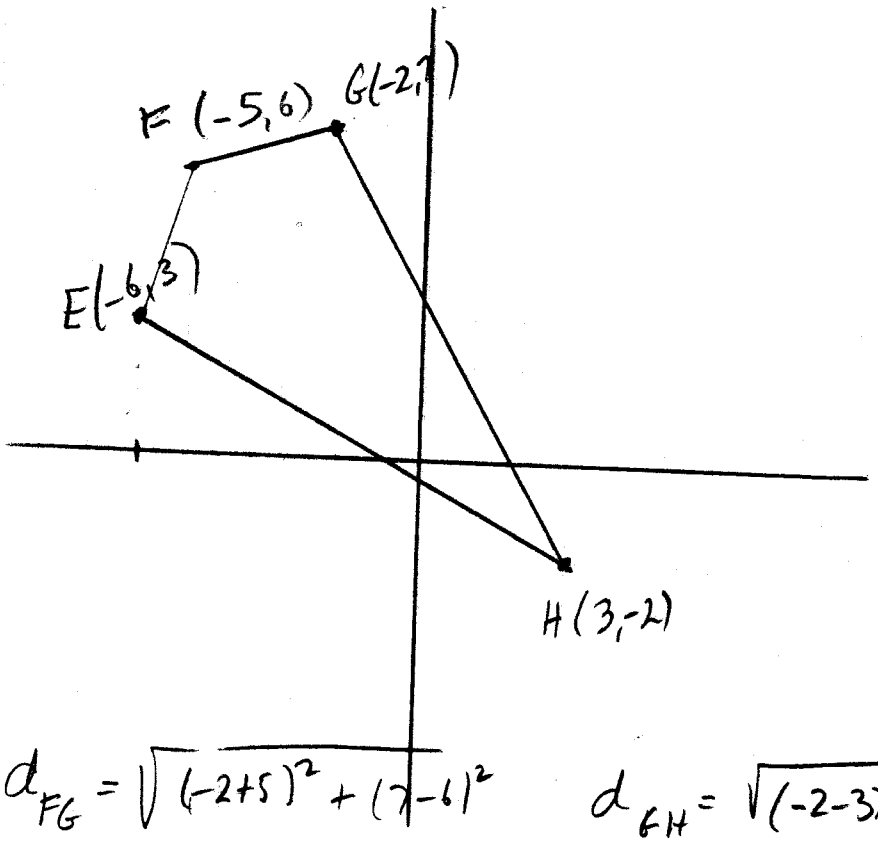
$d_{AD} = \sqrt{(-6-4)^2 + (1+1)^2}$

$= \sqrt{100+4}$

$= \sqrt{104}$

$\therefore$ adjacent sides CD + AD are not equal ABCD is not a kite + is an irregular quadrilateral

3.



$d_{FG} = \sqrt{(-2+5)^2 + (7-6)^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$

$d_{EF} = \sqrt{(-5+6)^2 + (6-3)^2}$

$= \sqrt{1+9}$

$= \sqrt{10}$

$\therefore d_{EF} = d_{FG}$

$d_{GH} = \sqrt{(-2-3)^2 + (7+2)^2}$

$= \sqrt{25+81}$

$= \sqrt{106}$

$d_{EH} = \sqrt{(-6-3)^2 + (3+2)^2}$

$= \sqrt{81+25}$

$= \sqrt{106}$

$\therefore d_{GH} = d_{EH}$

$\therefore$ Adjacent sides are equal but different in length

EFGH must be a kite