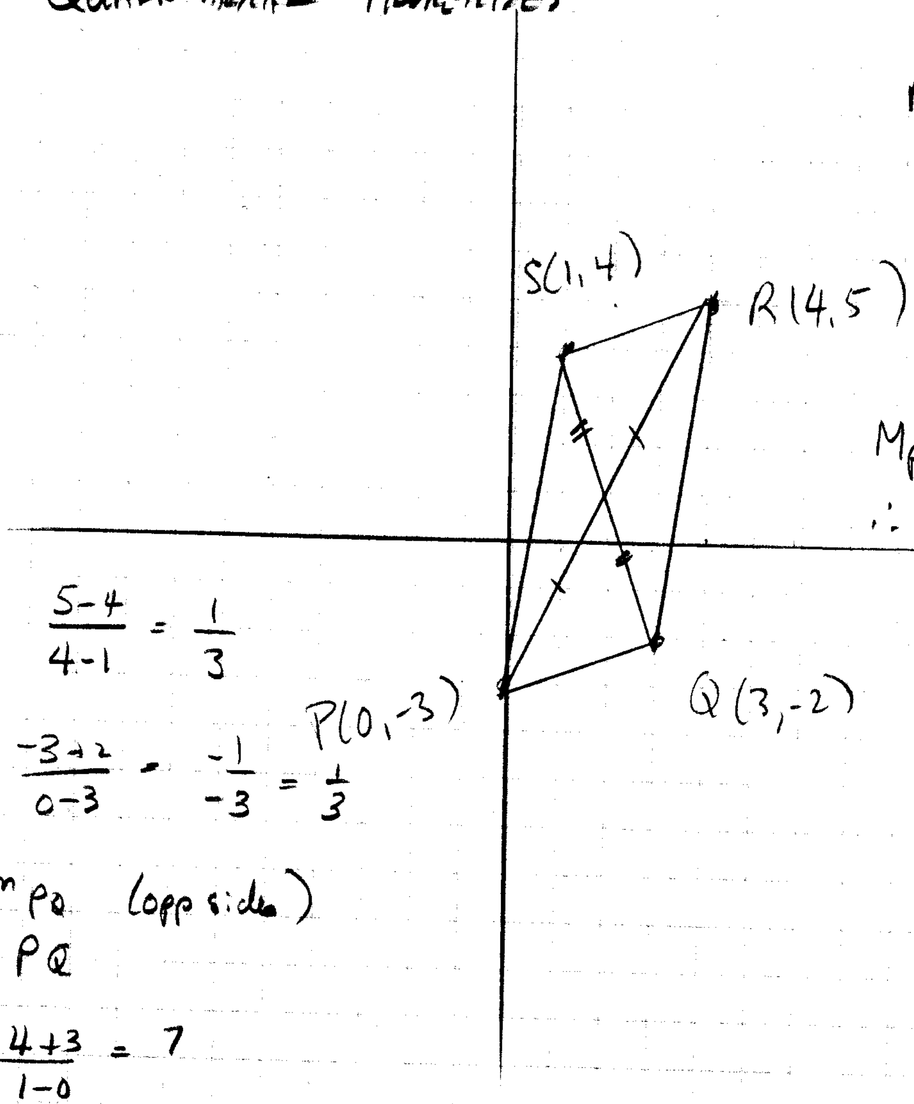
H1 Quadrilateral Properties

$M_{SQ} = \left(\frac{4}{2}, \frac{2}{2}\right)$
$= (2, 1)$

$M_{PR} = \left(\frac{4}{2}, \frac{5-3}{2}\right)$
$= (2, 1)$

$M_{PR} = M_{SQ}$!
$\therefore$ the diagonals do bisect each other.



$m_{RS} = \frac{5-4}{4-1} = \frac{1}{3}$

$m_{PQ} = \frac{-3+2}{0-3} = \frac{-1}{-3} = \frac{1}{3}$

$m_{RS} = m_{PQ}$ (opp sides)
$\therefore RS \parallel PQ$

$m_{SP} = \frac{4+3}{1-0} = 7$

$m_{RQ} = \frac{5+2}{4-1} = 7$

$m_{SP} = m_{RQ}$ (opp sides)

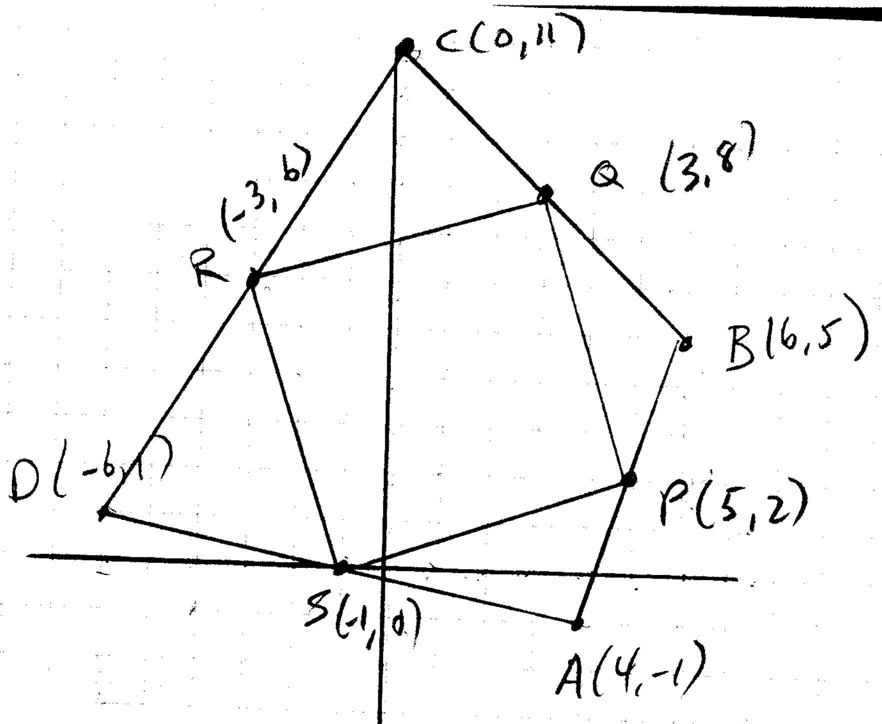
$\therefore SP \parallel RQ$ + PQRS is a llgm.
but slopes of adjacent sides are not neg. reciprocals.
$\therefore$ PQRS $\neq$ rectangle / square.

$d_{SR} = \sqrt{(5-4)^2 + (4-1)^2}$
$= \sqrt{1+9}$
$= \sqrt{10}$

$d_{SP} = \sqrt{(4+3)^2 + (1)^2}$
$= \sqrt{50}$

$\therefore$ lengths of adjacent sides are not equal
$\therefore$ PQRS is a parallelogram but not a rhombus, square or rectangle

2.



$m_{RS} = \frac{6-0}{-3+1} = \frac{6}{-2} = -3$

$m_{QP} = \frac{8-2}{3-5} = \frac{6}{-2} = -3$

$m_{RS} = m_{QP}$

$\therefore RS \parallel QP$

$m_{RQ} = \frac{8-6}{3+3} = \frac{2}{6} = \frac{1}{3}$

$m_{SP} = \frac{2-0}{5+1} = \frac{2}{6} = \frac{1}{3}$

$m_{RQ} = m_{SP}$

$\therefore RQ \parallel SP$ + PQRS is a $\parallel$gm.

$M_{AB} = \left(\frac{6+4}{2}, \frac{5-1}{2}\right) = (5,2) = P$

$M_{BC} = \left(\frac{6}{2}, \frac{16}{2}\right) = (3,8) = Q$

$M_{CD} = \left(\frac{-6}{2}, \frac{11+1}{2}\right) = (-3\ 6) = R$

$M_{DA} = \left(\frac{-6+4}{2}, \frac{1-1}{2}\right) = (-1, 0) = S$

What type of Quadrilateral is ABCD?

$m_{AB} = \frac{3}{1} = 3.$

$m_{CD} = \frac{11-1}{+6} = \frac{10}{6} = \frac{5}{3}$

$m_{BC} = \frac{11-5}{0-6} = \frac{6}{-6} = -1$

$m_{AD} = \frac{1+1}{-6-4} = \frac{2}{-10} = -\frac{1}{5}$

no sides are $\parallel$ or at 90°

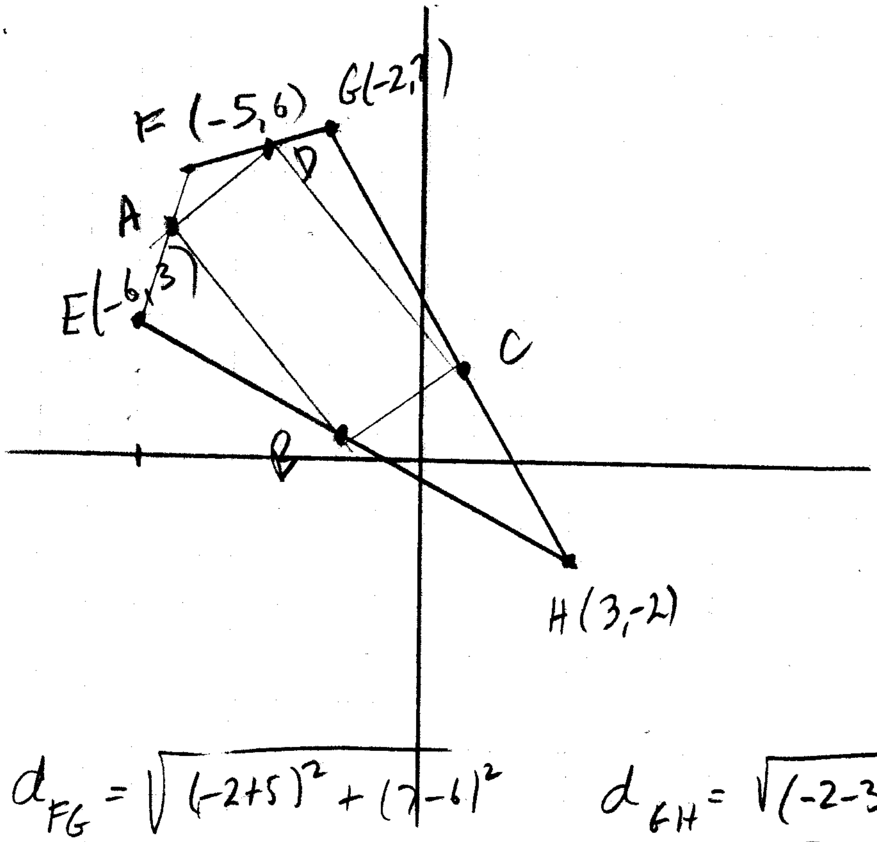
$\therefore$ ABCD is no type of $\parallel$gm or trapezoid but could be a kite.

$d_{CD} = \sqrt{(0+6)^2 + (11-1)^2}$
$= \sqrt{36+100}$
$= \sqrt{136}$

$d_{AD} = \sqrt{(-6-4)^2 + (1+1)^2}$
$= \sqrt{100+4}$
$= \sqrt{104}$

$\because$ adjacent sides CD + AD are not equal ABCD is not a kite + is an irregular quadrilateral

3.



$d_{FG} = \sqrt{(-2+5)^2 + (7-6)^2}$
$= \sqrt{9+1}$
$= \sqrt{10}$

$d_{GH} = \sqrt{(-2-3)^2 + (7+2)^2}$
$= \sqrt{25+81}$
$= \sqrt{106}$

$d_{EF} = \sqrt{(-5+6)^2 + (6-3)^2}$
$= \sqrt{1+9}$
$= \sqrt{10}$

$d_{EH} = \sqrt{(-6-3)^2 + (3+2)^2}$
$= \sqrt{81+25}$
$= \sqrt{106}$

$\therefore d_{EF} = d_{FG}$

$\therefore d_{GH} = d_{EH}$

$\therefore$ Adjacent sides are equal but different in length
EFGH must be a kite

cont'd.

#3. the Midpoint Shape.

$M_{EF} = \left(-\frac{11}{2}, \frac{9}{2}\right) = (-5.5, 4.5) = A$

$M_{EH} = \left(-\frac{3}{2}, \frac{1}{2}\right) = (-1.5, 0.5) = B$

$M_{HG} = \left(\frac{1}{2}, \frac{5}{2}\right) = (0.5, 2.5) = C$

$M_{FG} = \left(-\frac{7}{2}, \frac{13}{2}\right) = (-3.5, 6.5) = D$

$m_{AB} = \frac{4.5 - 0.5}{-5.5 + 1.5} = \frac{4}{-4} = -1$

$m_{BC} = \frac{0.5 - 2.5}{-1.5 - 0.5} = \frac{-2}{-2} = 1$

$BC \perp AB$.

$m_{AD} = \frac{6.5 - 4.5}{-3.5 + 5.5} = \frac{2}{2} = 1$

$m_{DC} = \frac{6.5 - 2.5}{-3.5 - 0.5} = \frac{4}{-4} = -1$

$AD \perp DC$ and $AB \parallel DC$ and $BC \parallel AD$

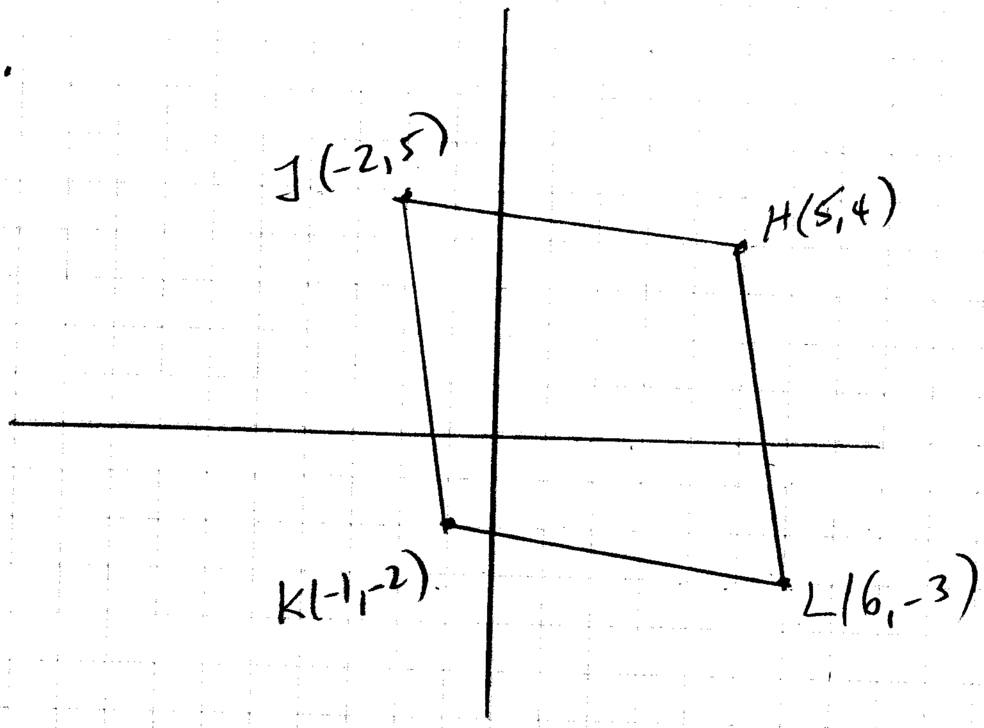
∴ opp sides are parallel and 1 pair of adj sides are at a 90° angle.

$d_{AB} = \sqrt{(-5.5+1.5)^2 + (4.5-0.5)^2} = \sqrt{4^2 + 4^2}$
$= \sqrt{32}$

$d_{BC} = \sqrt{(-1.5+3.5)^2 + (0.5-6.5)^2} = \sqrt{4 + 36}$
$= \sqrt{40}$

∴ ABCD is a rectangle.

4.



$m_{JH} = \dfrac{5-4}{-2-5} = \dfrac{-1}{-7}$

$m_{JK} = \dfrac{5+2}{-2+1} = \dfrac{7}{1} = -7$

$m_{KL} = \dfrac{-2+3}{-1-6} = \dfrac{-1}{7}$

$m_{HL} = \dfrac{4+3}{5-6} = -7$

$\therefore$ KL $\parallel$ JH (opp sides)

$\therefore$ JK $\parallel$ HL (opp sides)

$\because$ adj slopes are not negative reciprocals HJKL is a parallelogram but not a square or rectangle.

$d_{JH} = \sqrt{(-2-5)^2+(5-4)^2} = \sqrt{50}$

$d_{JK} = \sqrt{(-2+1)^2+(5+2)^2}$

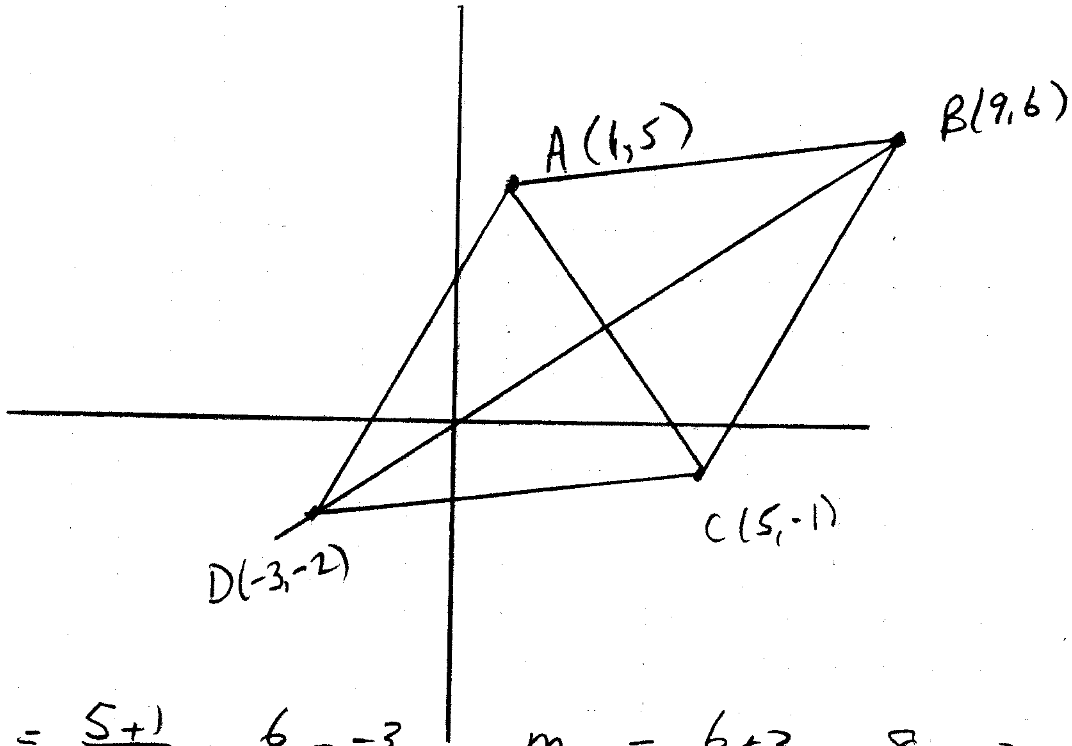
$= \sqrt{1^2+7^2}$

$= \sqrt{50}$

$\because$ adjacent sides are =, + opp sides are $\parallel$

HJKL is a rhombus.

5.



$m_{AC} = \frac{5+1}{1-5} = \frac{6}{-4} = \frac{-3}{2}$ $\quad m_{BD} = \frac{6+2}{9+3} = \frac{8}{12} = \frac{2}{3}$

$\because$ m_{AC} is the neg reciprocal of m_{BD}, $m_{BD} \perp m_{AC}$
+ the diagonals intersect $90°$

$M_{AC} = \left(\frac{1+5}{2}, \frac{5-1}{2}\right) = (3,2)$

$M_{BD} = \left(\frac{9-3}{2}, \frac{6-2}{2}\right) = (3,2)$

$\therefore$ the diagonals intersect at their shared midpoint at $90°$
and are $\perp$ bisectors of each other.

$m_{AB} = \frac{6-5}{9-1} = \frac{1}{8}$ $\quad m_{CD} = \frac{-1+2}{5+3} = \frac{1}{8}$ $\quad m_{BC} = \frac{6+1}{9-5} = \frac{7}{2}$

$d_{AD} = \sqrt{(1+3)^2+(5+2)^2} = \sqrt{16+49} = \sqrt{65}$ $\quad d_{AB} = \sqrt{(1-9)^2+(5-6)^2}$

$d_{BC} = \sqrt{(9-5)^2+(6+1)^2} = \sqrt{65}$ $\quad = \sqrt{8^2+1^2}$

$= \sqrt{65}$

$\because$ 1 pair of opp sides are $\parallel$ but not $\perp$ to the adj sides, + 3 sides
are equal ABCD must be a rhombus.

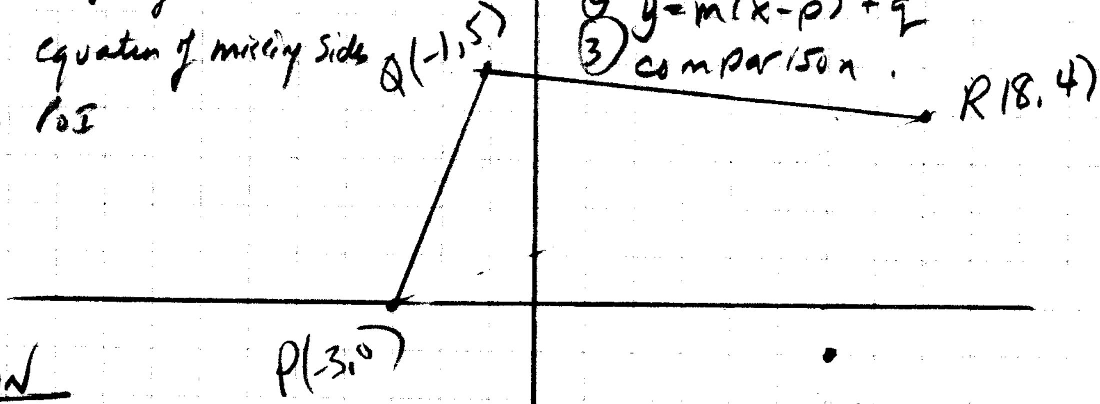
6.

STRATEGY

① Find slope of sides
② Find equation of missing side
③ Find POI

Tools

① slope formula.
② $y = m(x-p)+q$
③ comparison.



EXECUTION

①

$$m_{QP} = \frac{5-0}{-1+3} = \frac{5}{2}$$

$$\therefore\ m_{RS} = \frac{5}{2}.$$

$$m_{QR} = \frac{5-4}{-1-8} = \frac{1}{-9}$$

$$\therefore\ m_{PS} = -\frac{1}{9}$$

② RS

$$y = \frac{5}{2}(x-8)+4$$

$$y = \frac{5}{2}x - 20 + 4$$

$$y = \frac{5}{2}x - 16 \quad ①$$

PS

$$y = -\frac{1}{9}(x+3)$$

$$y = -\frac{1}{9}x - \frac{1}{3} \quad ②$$

③ POI

$$\frac{5}{2}x - 16 = -\frac{1}{9}x - \frac{1}{3}$$

$$45x - 288 = -2x - 6$$

$$47x = 282$$

$$x = 6$$

Sub $x = 6$ into ①

$$y = \frac{5}{2}(6) - 16$$

$$= -1$$

CONCLUSION

POI $= (6,-1)$ $\therefore$ the 4^{th} vertex is $(6,-1)$